

## Timed Coloured Petri Nets and Project Management Applications

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**Abstract:** Traditional techniques for project modelling have not adequately incorporated some factors that are essential for resource planning and management. This paper describes a new approach for project modelling that uses Timed Coloured Petri nets (TCPNs), to facilitate resource allocation in projects under constraints, commonly encountered in practice since TCPNs provide a powerful formalism for representing and analysing parallel systems. However, up to now, very little has been done to integrate this graphical and mathematical tool with the area of project management. TCPN models can be used to analyse interdependencies, criticality, substitution, conflicting resource priorities and variations in the availability of resources. This paper proposes a new model, demonstrating its usefulness for real-time activity scheduling in a resource-constrained project environment. The analysed case study regards the construction of an Italian highway.

**Keywords:** Timed Coloured Petri Nets, Modelling Tools and Techniques, Project Management, Resource-Constrained Project Scheduling Problem, Project Modelling

### 1. Introduction

The project implementation needs competencies from different areas of the business to be coordinated and synchronised. Many parameters exist for the success of any project, such as completion time, specific budget and technical features (Bevilacqua et al. 2015). Particularly, according to Avots (1969), factors affecting the project management success are several, they include the inadequate basis for the project and administration techniques misuse. In fact, the use of proper management techniques is a relevant requirement to execute a project successfully. Furthermore, the fluctuation in the economic, financial and environmental causes discriminates each project since unforeseen events are common during project progress (Anderson and Merna 2003). Thus, it is necessary to apply a rigorous methodology based on principles and systematic rules to manage complex projects.

For these reasons, the presented study aims at developing an approach for project modelling and managing, using powerful graphical and analytical tools: the Timed Coloured Petri Nets (TCPNs), concerning the System Dynamics (SD) research field. In particular, SD can allow managers to comprehend the project dynamics in order to model the inter-relationships among factors, and quantify their joined effects on a project (Williams 2000).

In this context, TCPNs offer several advantages to project managers such as the possibility to model a system taking into account numerous concurrent activities and conflicts.

Moreover, system deadlocks and bottlenecks can be identified.

The paper is structured as follow. Section 2 describes the TCPN methodology and some applications concisely. Section 3 summarises the adopted research approach, the modelling phase, and the case study description. Section 4 presents and discusses the empirical results about the case study. Finally, Section 5 discusses the findings of this study.

### 2. Modelling with TCPNs

The investigated literature highlights the significant interest on the project success. Murray, Bennett et al. (2009) studied the Public–Private Partnerships Critical Success Factor revising about 72 scientific papers published in 52 international journals. Scott-Young et al. (2008) research defines the guidelines for meeting the project requirements properly, organising the project activities. It is possible to find many other researches regarding traditional project modelling tools (Baker, 1974; Ahuja et al., 2014 and Pinedo 2012).

Project management standards propose tools for managing the project execution and the control phase. Nevertheless, it is important to underline that standards are largely used in the specific industry, and often, they do not support cross-industry and cross-organisational collaboration (Müller, Wald et al. 2012). For this reason, traditional management tools are not able to extend their application. On these grounds, many complex industrial

systems have been analysed recently using high-level Petri nets, including “colour”, “time” and “hierarchy” attributes. Mazzuto et al. (2012) identified a methodology, TCPN based, for designing and managing a Supply Chain (SC) and, simultaneously, for evaluating the performance of the production chain stakeholder. Zhu et al. (2004) applied a CPN to model activity interfaces and composite activity varieties, focusing on the internal states of all activity interfaces. Similarly, Chung (2011) illustrated the CPNs utility to model construction schedules, means a new scheduling method concerning the Critical Chain Scheduling (CPN-CCS) theory. Zhu et al. (2004) used TCPNs to express formalism for configuring an SC efficiently. They carried out a modelling tool for the supply chain configuration, able to assist businesses during their configuration. Shen et al. (2005) used TCPNs approach to propose a Concurrent Project Modelling Method in order to arrange the bank activities in a marketing division, allowing the analyst to consider parallelism and concurrency, synchronisation and resource sharing (Jeetendra et al. 2000).

**2.1 The TCPN structure**

In the context of PN, different methods for modelling have been realised, according to the complexity of the requirements. Generally, places represent conditions and results of actions (identified by the transition), and the information is only given by the system structure. In addition, Timed and Stochastic PNs (TPN, SPN) include probabilistic information about the considered system (Marsan, Balbo et al. 1991). For example, the stochastic timed Petri net model, designed by Bortolini et al. (2016), performs a scenarios analysis, taking in consideration an assembly line and the production and maintenance activity scheduling.

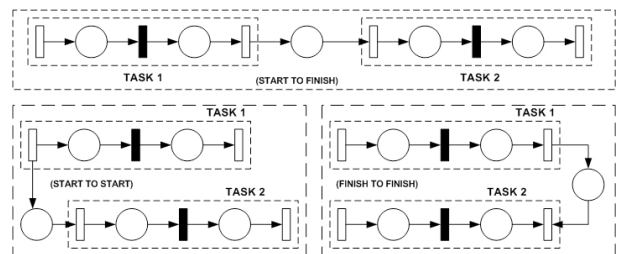
Interesting is the study of Kammoun et al. (2017) in which they proposes an optimal solution for a Flexible Manufacturing System (FMS) scheduling problems, subject to availability constraints, and based on Timed Petri Nets (TPNs). In particular, the realised mathematical model allows determining the optimal firing transitions sequence in order to minimise the total manufacturing time.

Finally, Coloured PNs (CPN) can be identified as an extension of TPCs. The colours’ presence makes CPNs/TCPNs the perfect tools for describing systems containing many interacting components and information (Jensen 1996). CPNs/TCPNs have been developed to provide mechanisms for accommodating the system changes. Graphically, white circles identify places; transitions are modelled using white (for untimed transition) or black rectangles (for timed transitions). Arcs join a place to a transition or vice versa, never among objects of identical nature. For more specification about CPNs/TCPNs, it is possible to refer to Jensen (1996).

**2.2 TCPN for project modelling**

The TCPN are able to model a system composed of many activities that take place concurrently and asynchronously. Particularly, it is suitable to model concurrences, conflicts, and system deadlocks. Moreover, the use of TCPNs allows checking the project health state, identifying possible activity delays. TCPNs can also model regenerating and rescheduling activities, paying attention to breakdowns and resource constraints. It is possible to represent graphically the project/process dynamic through subnets, places and transitions, simulating the entire system. The realised model is so able to represent resource interdependency, partial allocation, substitution and mutual exclusivity.

Behavioural properties (reachability and boundedness) allow to model complex systems with resources constraints. Thus, higher-level Petri Nets reduce the network graphical size, avoiding the complicated inclusion of dummy activities and making easier to model and represent precedence relationships, as shown in Figure 1.



**Figure 1:** Examples of representations of project management relationships between activities

Several properties such as transition name, firing delay and firing weights, can be related to a transition, but the “transition priority” is the most relevant one, to establish different priorities among concurrent transitions. According to Mejía et al. (2016), Petri Nets can be considered suitable to model the features of a project, taking into account the resource constraints. Only a small number of studies, though, implement PNs for project scheduling and control.

Rinke et al. (2017) demonstrates how hierarchical and modular modelling of construction processes (high-level Petri nets) carries a base for real time quality evaluation and re-planning on construction site.

**3. The research framework**

The realised TCPN model can be described by the multi-step procedure reported in Figure 2.

As shown in Figure 2, it is possible to highlight that the starting point is the Work Breakdown Structure (WBS) phase.

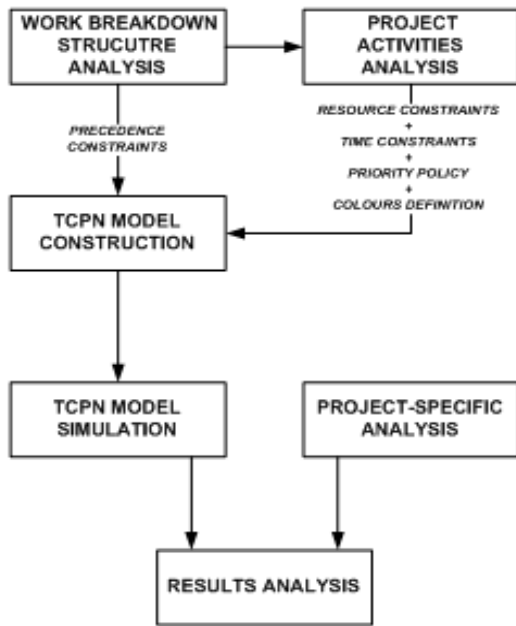


Figure 2: The research approach.

The WBS describes and clusters a project's discrete work elements, defining the final objective and its deliverables. The "Project activities analysis" studies all the activities to identify all of the relevant features (resource request, duration and priority). The results are a specification of resources and time constraints, in addition to the priority policy. Hence, all these components contribute to determining the colours to be used in the model. Thus, in the "TCPN Model Construction" phase, the application models the entire project structure, taking into account the mentioned constraints. In particular, possible delays on each activity can be defined and selected by the project manager.

Then, during the "Simulation" phase, the system implements different "activity priority" values in order to allow managers to define the less critical scheduling, evaluating different possible scenarios and solutions. Finally, in the "Results Analysis" phase the simulation results are compared with the project requirements (the output of the "Project Specific Analysis"). Specifically, the model can implement several literature-based tie-breaking rules in order to manage low resource availability (Browning et al., 2010; Fündeling et al., 2010; Xu et al., 2008; Buddhakulsomsiri et al., 2007; Brucker et al., 1999) such as "minimum Slack time", "longest task execution" and "Greatest Ranked Positional Weight".

### 3.1 The case study

The proposed case study regards a public work inherent the adjustment of an existing road layout. A complete contract analysis has been developed to evaluate an alternative planning, compared with the hypothesis made by the provincial administration, and concerning a total duration of 740 working days. The analysis takes into account only a street line of length equal to 4160 kilometres (see Figure 3).



Figure 3: The considered case study

Five macro-phases have been identified and reported in Table 1.

Each macro-phase has been analysed and decomposed in activities. In addition, the same analysis has been carried out for each of them in order to identify constraints and required resources. Figure 4 shows the relationships between the macro-phases.

Thus, the project schedule has been analysed implementing the principles of Critical Path Method (CPM) and Critical Chain Project Management (CCPM), in order to compare the results.

Table 1: Description of the project macro-phases

Phase ID	Duration (dd)	Resources (%)	
		Workers	Mechanical Means
PH1	273	20	80
PH2	148	35	65
PH3	168	35	65
PH4	77	75	25
PH5	70	82	18

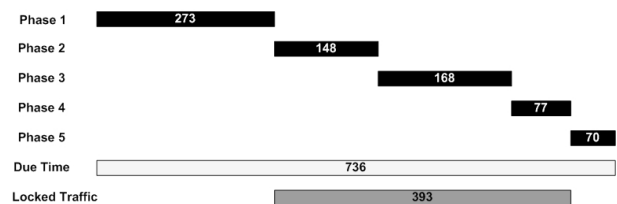


Figure 4: The relationships between the macro-phases

**Table 2:** Colours definition for the TCPN model.

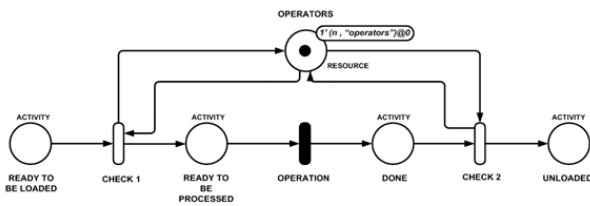
COLOUR NAME	DEFINITION
Sequence	$ActDone \times ActToDo$
Duration	$mind \times ModeD \times MaxD$
Order	Order
Resources	$ResType \times Quant$
Activity	$Sequence \times Duration \times Order \times Resources$
Resources	Resources

**3.2 The basic model**

The system basic unit, used to model one project activity is shown in Figure 5. The colours defined to manage the TCPN system (which are numbered from 1 to 4) can be described as in the following:

1. a couple of values identify the activity and its successors: (ActDone, ActToDo);
2. a Beta probability density distribution, identified by minimum, mode and maximum, for the activity duration: (MinD, ModeD, MaxD), according to PERT approach;
3. activity precedence constraints allow to draw the project graph: (Order);
4. two parameters to identify the activity resources, and represent the resource type and its availability: (ResType, Quant).

Table 2 shows the TCPN elementary and structured colours, referring to the definition.

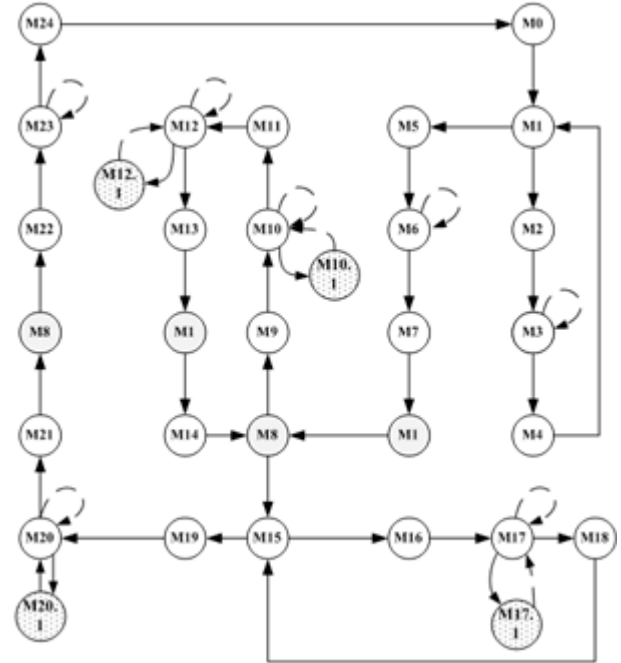


**Figure 5:** Project activity basic model.

**4. Results and Discussion**

After the project activity modelling, some delay causes have been analysed to be subsequently modelled. The rain has been identified as the most frequent. Specifically, the investigation of the historical period ranging between 1961 and 2006 allowed to quantify the annual rainy days, for the considered province, equal to 80 days. It means that, considering the project evolution, a delay equal to 1.54 days for week (one year is composed of 52 weeks) has been taken in consideration.

After the completion of the modelling phase, the entire project has been also simulated using the traditional PERT/CPM approach, in order to define the best activities scheduling and identify the best resources distribution during the whole period. As result, the macro-phases 2, 3 and 4 have been ranked as the most critical in terms of resources usage.

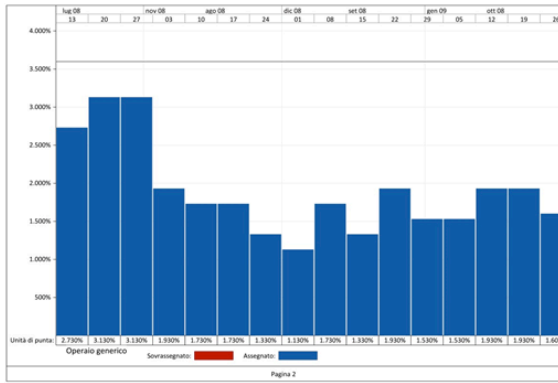


**Figure 6:** The project marking tree after simulation

In fact, one of the most important application of the proposed tool is the possibility to identify the potential project bottlenecks by analysing the project-marking tree. Indeed, the marking tree is a project manager method to compare and highlight differences between the project planned and the real or simulated schedule when possible critical situations occur. Figure 6 shows a detailed summary for the explained critical situation. The dotted circles identify the conditions in which there is an insufficient availability of the “workers” resource.

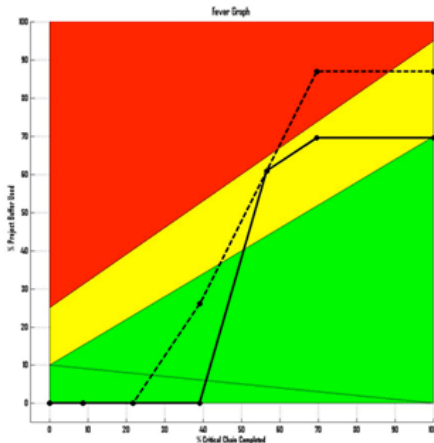
Through the simulation, and according to different priority rules, the best solution considers a partial overlap of consecutive workmanships, if technically feasible. Figure 7 shows an excerpt of the resources distribution during the whole project evolution. In particular, the esteemed duration, obtained from the simulation, is equal to 729 days over the 740 days considered by the provincial administration.

According to the application of the CCPM methodology, the duration of the whole project execution has been esteemed equal to 631 days, conversely to the 736 of the CMP schedule with a reduction approximately equal to 15%. The entire critical chain is composed of 96 activities over the 279 that compound the project. An amount of 46 feeding buffers has been planned for a total duration of 276 working days, and a project buffer equal to 82 days.



**Figure 7:** The resources distribution during the project evolution

Moreover, the realised tool gives managers the possibility to control the buffer consumption, using the fever graph, and analyse the effects of specific corrective actions, to reduce delay in the project progress. In addition this tool explores a rarely investigated area, such as the project reaction to critical tasks length reduction. Figure 8 shows the fever graph. Dashed line represents the project status in presence of delay without considering any correction. On the contrary, the continuous line identifies the project status in which, in presence of delay, managers conduct proper corrective actions. It is relevant to highlight that the tool is able to implement corrective actions in an automatic way, defining the best one.



**Figure 8:** the effects of the corrective action on the project scheduling

**5. Conclusion**

The intent of this research is to underline the benefits of TCPN in the project management area. The proposed TCPNs tool gathers reliable information both for project scheduling and controlling. The described structure consents to model easily the project using an "object-oriented" simulator, able to achieve scenario analyses. Additionally, the project simulation can yield lots of information concerning the system performance (such as

queue length, response time and buffers consumption) to be processed for a better project scheduling and controlling. In further detail, this tool allows “what if” scenarios analysis during the project execution, collecting relevant data for an in-depth decision-making process.

Due to the approach complexity, not easy to understand without any TCPN knowledge, future work will be focused on several aspects. In terms of modelling and scheduling, the efforts will be dedicated to refine the Timed Coloured Petri Net model in order to simplify the model construction. Other algorithms will be implemented and tested, taking into account several and different objective functions. Finally, it will be investigated the impact on the project success of workers with different skills. Concerning the practical usage, a software prototype, easy to use, will be designed to allow managers to calculate their schedules, subject to resource constraints and costs, gathering data that are more realistic. Finally, future development could involve a model extension to the multi-project environment, assisting project managers to work in a systemic view for an efficient resource exploitation.

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