

The distribution planning process in a supply chain with multiple transportation strategies

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Abstract: This paper compares different approaches to solve the distribution planning process problem with distribution strategy choice in supply chains, encompassing several production plants and several regional warehouses that fulfill a set of retail customers with several references. The problem has been tackled considering two different perspectives: the former is more empirical, providing a heuristic solution (Empirical model) of the problem, while the latter is based on a mixed integer linear programming model (Analytical model). The paper discusses the computational results obtained by applying the aforementioned approaches in terms of costs, optimality gap and computational time to the food supply chain of an Italian subsidiary of a German group encompassing 2 production plants located in Germany, 2 regional warehouses situated in Italy, that fulfill 200 retail customers with 19 references. In addition to the computational results that provide a comparison among the solution applied by the company (Rule of thumb), the Empirical model and the Analytical model, managerial insights are underlined, in terms of applicability of the best approach within the specific company context.

Keywords: Supply chain; Distribution planning process; Distribution strategy; Transportation mode choice

1. Introduction

In this paper, we study a supply chain delivering products directly from production plants to customers and indirectly, first from production plants to warehouses and then from there to the customers. In particular, we consider an Italian subsidiary of a German group, which manages three different product categories, encompassing dry references (from ingredients of pastry prepared for cakes), cold references (puddings, panna cotta and yogurt) and frozen references (including 19 tastes of frozen pizza). This paper focuses on the distribution planning process of the frozen references. The aim of the work is to provide solution approaches to determine the distribution strategy that minimizes the total cost of distribution activities.

2. Objectives

The main research objective is to show the value of using more and more sophisticated optimization tools in the solution of real cases and to provide managerial insights and guidelines for the selection of the best approach depending on the company context. We propose and compare two alternative approaches that can be used to solve the problem of the distribution strategy choice in a supply chain, evaluating their performances compared to the actual solution implemented (in the following, *Rule of thumb*): the *Empirical model* (EM) and the *Analytical model* (AM). The former is an evolution of the rigid and pragmatic Rule of thumb adopted by the Company to manage the distribution & transportation planning process. This approach provides a heuristic solution of the

problem that requires limited time to compute it. The Analytical model is based on the solution of a mixed integer linear programming model. This approach provides an optimal solution (or the best solution in a given time limit) and a lower bound on the optimal cost, which can be used to evaluate the solutions provided by the Rule of thumb and by the EM. The comparison of the two approaches with the solution obtained by applying the Rule of thumb in a large set of real instances allows us to have a clear picture about which approach best fits the real case and to provide general managerial insights in the design of distribution planning systems in supply chains with distribution strategy choice.

3. Distribution planning process

A relevant classification of the possible approaches to distribution planning process is based on the planning horizon: from a wider to a closer perspective, it is possible to consider strategic, tactical and operational approaches. Sabri and Beamon (2000), SteadieSeifi et al. (2014) and Wei et al. (2016) can help understanding the implication of this classification upon this work. We do not solve strategic and tactical problems. In fact, the supply chain structure and configuration is taken into account as a matter of fact, and we do not solve problems related to decisions on the infrastructures and networks. Our main goal is to determine, at an operational level, how and when move products from the production plants to the warehouses and from the production plants and the warehouses to the customers, with the aim to minimize the total cost. In fact, operational planning deals with real-

time planning for orders, and reaction and adjustment to any kind of disturbance that are not explicitly addressed at strategic and tactical levels (StadieSeifi et al., 2014). According to this definition, we propose two operational level approaches: i). an EM, that can be run to have a solution that can be computed incrementally over time, after the continuous receiving of the customers’ orders; ii). an Analytical model, that requires a more consolidated set of input data to be run (i.e., the overall weekly orders’ portfolio has to be available at the beginning of the planning horizon).

Since in our problem the key decision is related to the choice of different available distribution strategies, that can be based on different transportation modes, is fundamental to review the extant literature containing elements of transportation mode choice and selection of the most effective distribution strategies.

We focus on the most related mathematical treatments of transportation mode choice. For a general survey documenting industry challenges, we refer the reader to Meixell and Norbis (2008).

There is a thread of research which incorporates vehicle routing with mode choice (typically a choice between delivery on routes executed by an internal fleet or delivery via an external carrier). Due to the complexity of the resulting problem (an enriched VRP), this work has focused on the development of heuristic approaches. Côté and Potvin (2009) and Potvin and Naud (2011) are recent works along this meme. The vehicle routing literature also includes models that incorporate a cross-dock. Representative work in this area includes Wen et al. (2009) and Tarantilis (2013). These models are concerned with sequencing the loads and the dependencies between the vehicles arriving to and departing from the cross-dock. Due to the complexity of the problem, most studies focus on the development of heuristics. The effect of mode on shipment size has also been examined. Hall (1985) considers a single supplier and single customer to simultaneously determine the optimal mode and shipment size. In related work, Archetti et al. (2014) study lot-sizing in the presence of a transportation cost based on tiered echelons. Bertazzi and Ohlmann (2014) compare the performance of the various distribution policies, based on direct shipping, transit point and 2-routing.

4. Description of as-is context

The problem takes place in a supply chain through which frozen references are delivered from German production plants to Italian customers. The supply chain is composed as follows:

1. At the first level, there are two German production plants;
2. At the second level, there are two Italian warehouses;
3. At the third level there are almost 200 customers.

Due to this supply chain configuration, there are two possible distribution policies to serve customers, which imply different independent distribution strategies:

- *Direct shipping*: the customers’ demand is served directly from the German warehouses. Depending on customer location, it could be served either in one day (clients in the north part of Italy) or in two days (clients in the center and in the south part of Italy). Trucks starting from the German warehouses must be at full load. According to this constraint, there are two different direct delivery ways:
 - *One-step direct shipping*: when customer requires exactly the capacity of one or more trucks (**33** pallets or multiples), the delivery provides only one unloading of goods at the retail store;
 - *Two-steps direct shipping*: when customer does not require a full load truck, the Company saturates the truck capacity with other references used to replenish the Italian warehouses. In this case, in addition to the customer unload, another one is planned to one of the two Italian warehouses.
- *Indirect shipping*: the demand is served through the Italian warehouses (depending on the customer location). Every customer is replenished in one day.

5. Formal description of as-is context

We consider a supply chain composed of a set $I = \{1, 2, \dots, |I|\}$ of production plants, a set $K = \{1, 2, \dots, |K|\}$ of warehouses and a set $J = \{1, 2, \dots, |J|\}$ of customers. A set $S = \{1, 2, \dots, |S|\}$ of products is produced at the plants and delivered to the customers, either directly or indirectly through the warehouses. Shipments can be performed on the set of days $T = \{1, 2, \dots, |T|\}$. Let $\Omega \subset T$ be the set of days in which shipments cannot be performed from the production plants (e.g. on Sunday). For each product $s \in S$, we are given the initial inventory level \bar{B}_{0si} at each production plant $i \in I$, the maximum inventory level $\bar{B}_{|T|si}$ on the last day at each production plant i , the initial inventory level \bar{I}_{0sk} at each warehouse $k \in K$, the quantity q_{tsi} produced at each production plant $i \in I$ on each day $t \in T$ and the demand d_{tsj} by each customer $j \in J$ on each day $t \in T$. The demand on each day t cannot be delivered to the customer in advance. A fleet composed of V vehicles is available at the production plants. Each vehicle has a transportation capacity of Q pallets and implies a fixed transportation cost c (corresponding to the full load). These vehicles can be used on each day either for 1) a *One-step direct shipping* to a customer; 2) a *Two-steps direct shipping* to a customer; 3) a *Warehouse replenishment*. Let D_j be a parameter equal to 1 if customer j can be served by a one-step direct shipping and equal to 0 otherwise and W_{kj} be a parameter equal to 1 if warehouse k can serve customer j and 0 otherwise. The delivery time for a *Warehouse replenishment* from production plant i to warehouse k is L_{ik}^R . The delivery time for a *One-step direct*

shipping from the production plant i to the customer j is L_{ij}^D days, while it is L_{ikj}^{2D} for a *Two-steps direct shipping* from the production plant i to the warehouse k and then from k to the customer j . Shipments from the warehouses to the customers are outsourced. The transportation cost to send p pallets from warehouse k to customer j is f_{kjp} , where f_{kjp} is a less than proportional increasing function in the number of pallets $p \in P = \{1, 2, \dots, |P|\}$. The delivery time from the warehouse k to the customer j is L_{kj}^W days. A unit inventory cost h_k is charged for each positive inventory level at each warehouse $k \in K$ at the end of each day $t \in T$. A unit handling cost r is paid for each pallet sent from the warehouses to the customers. The problem is to determine:

1. For each demand d_{tsj} , the distribution strategy to use (*One-step direct shipping*, *Two-steps direct shipping*, *Indirect shipping*).
2. For each day t , the replenishment quantity of each product from each production plant to each warehouse to load on the vehicles performing a *Two-steps direct shipping*.
3. For each day t , the replenishment quantity of each product from each production plant to each warehouse.

Considering the constraints related to the impossibility to deliver the products in advance to the customers, the aim is to minimize the overall total cost and not consider other criterion such as responsiveness and customer service level (Liu and Papageorgiou, 2013). The objective function is given by the sum of the transportation cost for *Warehouse replenishment*, *One-step direct shipping*, *Two-steps direct shipping* and *Indirect shipping*, the inventory cost and the handling cost at the warehouses.

6. Solution approaches

6.1 Rule of thumb

The Rule of thumbs, exploited by company’s employees to identify one of the possible shipping modes to serve customers, counts 7 major assessment stages: for every one of these, the internal operators have to evaluate a specific parameter, and only if it satisfies the threshold value, the evaluation process will continue. Following are summarized the evaluation phases, with the description of the value parameters, namely the thresholds that permit to discriminate among direct or indirect delivery.

1. Order quantity ≥ 10 pallets
2. Delivery lead time ≥ 5 days from the order date
3. Number of pallets = multiple of truck capacity
4. Geographic localization not islands (Sicily and Sardinia)
5. Inventory level at production plants \geq reference order quantity
6. Date of delivery: not Tuesday
7. Trucks at production plants: available

Indeed, German headquarters imposed for each product an upper stock limit (in terms of number of pallet), in

order to maintain the lowest possible the value of products stored within the plants. This constraint often pushed Company’s personnel to arrange a replenishment shipping to Italian warehouses in order to not overcome this threshold.

6.2 Empirical model (EM)

The EM is an evolution of the Rule of thumb manually adopted by the Company’s employees. First, the Rule of thumb is implemented by using a programming language to make it completely automatic. According to the case context, only one parameter (the order quantity) is under the control of the Company, while the remaining six are exogenous, imposed by the German headquarters. The EM allows us to select the best order quantity as follows:

- Exactly replicating the Company approach and maintaining the pallet order quantity threshold constant over a year;
- Enhancing the flexibility of this model adapting the Rule of thumb applied by the Company: in particular, the period of validity of the order quantity parameter is narrowed to a week instead of a year. In this configuration, each week of the year becomes a different instance in which a What-if analysis is performed, identifying the order quantity threshold optimal value for that week. This variant is referred to as the Empirical model with flexible threshold (EMFT).

The EM considers the same constraints and the same variables exploited in the AM, described in Appendix 1, due to the complexity and the length of the model itself.

7. Instances and main results

As already described, the considered supply chain is composed of 2 German production plants, 2 Italian warehouses and 206 customers, all located in Italy. The complete dataset encompasses 6,000 orders, 22,000 order lines and 36,000 pallets delivered to customers in one year.

It has to be noticed that the EM is implemented in Visual Basic for Applications, while the mixed integer linear programming model in the Analytical one is solved by using CPLEX.

In order to provide a set of instances, the available data are segmented into weekly periods, thanks to which we obtain 52 instances. The main data required to run all the models are presented in Appendix 2. Table 1 shows the comparison, in terms of costs, optimality gap and computational time of the solution of the Rule of thumb used by the Company, the EM, the EMFT and the optimal solution of the AM with time limit of 4 minutes and the one of the AM with time limit of 60 minutes.

Table 1: Comparison of Rule of thumb, Empirical model and Analytical model

	<i>Rule of thumb (As-Is)</i>	<i>Empirical model (EM)</i>	<i>Empirical model with flexible threshold (EMFT)</i>	<i>Analytical model (AM) (4 min)</i>	<i>Analytical model (AM) (60 min)</i>
Order quantity threshold	Fixed = 10	Fixed = 23	Variable once a week	-	-
<i>Replenishment cost</i>	€ 49,071.00	€ 48,121.00	€ 42,487.00	€ 246,00	€ 316,00
<i>Direct shipping cost</i>	€ 945.00	€ 1,145.00	€ 1,145.00	€ 1,089.00	€ 808.00
<i>Two-steps direct shipping cost</i>	€ 3,925.00	€ 2,374.00	€ 2,374.00	€ 39,643.00	€ 39,854.00
<i>Indirect shipping cost</i>	€ 22,424.00	€ 21,061.00	€ 21,061.00	€ 15,480.00	€ 15,441.00
<i>Inventory cost</i>	€ 8,200.00	€ 8,002.00	€ 7,994.00	€ 7,611.00	€ 7,609.00
<i>Handling cost</i>	€ 3,229.00	€ 3,343.00	€ 3,327.00	€ 2,863.00	€ 2,859.00
Total cost	€ 87,794.00	€ 79,468.00	€ 78,375.00	€ 66,932.00	€ 66,886.00
Optimality gap (%)	37.90	25.90	23.90	4.81	4.75
Computational time (seconds)	-	215	8,740	240	3,475

Table 1 shows the comparison, in terms of costs, optimality gap and computational time of the solution of the Rule of thumb used by the Company, the solution of the EM, the solution of the EMFT, the optimal solution of the AM with time limit of 4 minutes and the one of the AM with time limit of 60 minutes. In the Rule of thumb, a threshold of 10 pallets is always used for Direct shipping (as aforementioned, the Company sets this parameter once a year, exploiting the same value for all the weekly instances). In the EM, the threshold that minimized the total cost in the What-if analysis (23 pallets) is used, while in the EMFT, we use in any instance the order quantity threshold that minimizes the total cost in that instance. Finally, in the Analytical model no threshold is used, ensuring a higher level of flexibility due to the possibility to identify the optimal value for each day. The AM is solved first with a time limit of 4 minutes to compare the optimal solution obtained within this time limit with the solution of the EM that requires about 4 minutes on average. Then, the AM is solved with a time limit of 60 minutes to evaluate the improvement in the optimal solution if more computational time is allowed.

We first compare the solutions in terms of total cost and optimality gap. The results show that a reduction in the total cost is obtained by the Empirical model with respect to the Rule of thumb solution: the optimality gap of the EM is about 12% lower than the one of the Rule of thumb. Although the reduction seems not very large, this corresponds to a saving of about 8,300 euros per week

and therefore to about 400,000 euros per year. Instead, allowing a flexible threshold is not effective: the optimality gap of the EMFT is just 2% lower than the one of the Empirical model. By taking into account that the computational time of the EMFT is about 40 times the one of the EM and that managing a flexible policy is much more complex than managing a rigid one, we can conclude that the Empirical model dominates the one with flexible threshold. The AM with time limit equal to 4 minutes provides a significant improvement, ensuring to reach an optimality gap of about 5%. This corresponds to a saving of about 20,500 euros per week and therefore to about 1,000,000 euros per year. This is mainly due to the fact that, in the AM, the solution is computed by taking into account the overall orders’ portfolio of the week, instead of evaluating individual orders on the base of an iterative procedure, as in the EM. Moreover, this solution is an optimal solution in a given time limit, instead of just a heuristic solution. Instead, increasing the time limit from 4 minutes to 60 minutes is not effective, as the reduction in the total cost is less than 0.1%.

This first comparison allows us to conclude that, if the overall orders’ portfolio of the week is available at the beginning of the week (before computing the solution) and if managers agree in using a very flexible solution (that can provide a different value of order quantity parameter each day), the AM should be used. In the opposite case, the EM can be more appropriate, to have a solution that can be computed incrementally over time (when new orders are placed) and to have a more rigid (and therefore, simpler) solution to implement. In this

case, the total cost of the solution obtained by applying the AM can be used as a lower bound

8. Managerial insights

In the previous section the benefits achievable by the Analytical model with respect to the Empirical one in terms of economical savings (also imposing the same time limit) have been clearly underlined. This does not imply that the AM should be applied in any case, as the best solution approach of this problem. In fact, from a managerial point of view, there are some relevant advantages in the application of the EM, that justify its development even considering the lower performances in terms of economical savings.

First of all, it has to be noticed that the formulation of the mathematical model behind the AM required a lot of effort and time to be able to take into account the aspects exploited by the EM, while, in order to answer to the main Company's requirements, the EM was built in a short time, ensuring a validated and proved solution to the problem.

Another advantage of the EM concerns the possibility to test the impact of different parameters at the same time on the solution: the model is completely parametric. This process is subjected to several constraints imposed by German headquarters (related for example to the maximum inventory level of the references stored at production plants), that are not proved to be optimal or the best possible. The Company can easily run the EM, change the parameter value and identify any further levers for improvement for the entire distribution & transportation process. Instead, the AM manifests lower flexibility: it should be at least partially reformulated in order to test other parameters than the order quantity.

Moreover, as already cited before, from the technical standpoint, the rigid solution provided by the EM is more manageable than the flexible solution provided by the AM. There are at least three different aspects that lead to this statement. The first is related to the input data needed by the two models to be able to give a solution (the EM needs only the orders to be fulfilled in the next 3 days, while the Analytical one at least needs orders to be fulfilled in an entire week to be able to properly manage Warehouse replenishment). The second concerns the implementation/integration of the model within the company's informatics system: in order to automate the transportation mode choice evaluation process for each order received, the model should be integrated with the ERP used by the Company. The third is related to the increase of the computational time when the time horizon increases. The EM can manage any time horizon, as it works incrementally over time, while the AM could be extremely difficult to be optimally solved when the time horizon significantly increases.

On the other hand, the AM, even if the solution provided by the model is not applied, is useful from the managerial point of view to be able to evaluate how good is the solution applied by the Company or the solution provided by the Empirical model.

9. Conclusion

In this paper we compared two different approaches to solve the distribution planning process with distribution strategy choice in supply chains. We applied these models to the real case of a food supply chain encompassing 2 production plants located in Germany and 2 regional warehouses situated in Italy, that fulfil 200 retail customers. One of these models is more empirical, providing a heuristic solution (Empirical model) of the problem, obtained as an improvement of the experiential approach adopted by the Company itself (Rule of thumb). This approach can be used even in real time and can be easily modified by managers to evaluate variants of distribution policies. The other one is based on a mixed integer linear programming model (Analytical model). This second approach provides a more flexible solution, a large saving in the total cost and a benchmark to evaluate the Rule of thumb and the Empirical model, but requires the use of more sophisticated optimization tools. From a managerial point of view, we provided general guidelines to select the best approach and we showed the value and the “managerial cost” of introducing more and more sophisticated optimization tools in the solution of complex real cases.

This paper shows very well the trap in which many companies fall: often, they have to choose between rigorous approaches that lead to optimal solutions and more practical ones, that generate heuristic solutions. One of the main limitations of these work concerns the sequential creation of the different models: in fact, the EM has been defined in 4-5 weeks, while the AM has required almost 2 months to be completed and tested, considering that the problem can be classified as NP hard. This has led the company to dispose very quickly of the EM, to test and to use it till the AM has been completed. This approach has promptly generated the Empirical model's appreciations, narrowing the interest in the Analytical one.

Future research developments are devoted to the possible implementation of the Analytical model in the IT systems of the company, in order to provide an integrated solution that also the most operative employees may manage.

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Appendix A

Analytical problem formulation

In this section, we formulate the mixed integer linear programming model behind the *Analytical model*. Let us introduce the following decision variables:

1. x_{tsik}^R : non-negative continuous variable representing the quantity of product s sent from the production plant i to replenish the warehouse k on day t .
2. x_{tsij}^D : non-negative continuous variable representing the quantity of product s sent by *One-step direct shipping* from the production plant i to the customer j on day t .
3. x_{tsikj}^{2DR} : non-negative continuous variable representing the quantity of product s sent from the production plant i to replenish the warehouse k on a vehicle performing a *Two-steps direct shipping* to the customer j on day t .
4. x_{tsikj}^{2D} : non-negative continuous variable representing the quantity of product s sent by *Two-steps direct shipping*

from the production plant i to the customer j on day t on the vehicle visiting warehouse k .

5. x_{tskj}^W : non-negative continuous variable representing the quantity of product s sent from the warehouse k to the customer j on day t .
6. y_{tik}^R : non-negative integer variable representing the number of vehicles used for a *Warehouse replenishment* from the production plant i to the warehouse k on day t .
7. y_{tij}^D : non-negative integer variable representing the number of vehicles used for a *One-step direct shipping* from production plant i to customer j on day t .
8. y_{tikj}^{2D} : non-negative integer variable representing the number of vehicles used for a *Two-steps direct shipping* from the production plant i to the customer j through the warehouse k on day t .
9. z_{tkjp} : binary variable equal to 1 if p pallets are sent from the warehouse k to the customer j on day t and 0 otherwise.

We also introduce the non-negative continuous variables B_{tsi} and I_{tsk} representing the inventory level of product s at production plant i on day t and the inventory level of product s at warehouse k on day t , respectively.

The model can be formulated as follows:

1) Objective function:

The total cost, given by the sum of the transportation cost for *Warehouse replenishment*, *One-step direct shipping*, *Two-steps direct shipping* and *Indirect shipping*, the inventory cost and the handling cost at the warehouses, is minimized:

$$\begin{aligned} & \min \sum_{t \in T} \sum_{i \in I} \sum_{k \in K} c y_{tik}^R + \\ & \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} c y_{tij}^D + \sum_{t \in T} \sum_{i \in I} \sum_{k \in K} \sum_{j \in J} c y_{tikj}^{2D} \\ & + \sum_{t \in T} \sum_{k \in K} \sum_{j \in J} \sum_{p \in P} f_{kjp} z_{tkjp} + \\ & \sum_{t \in T} \sum_{s \in S} \sum_{k \in K} h_k I_{tsk} + r \sum_{t \in T} \sum_{s \in S} \sum_{k \in K} \sum_{j \in J} x_{tskj}^W \end{aligned}$$

2) Demand constraints:

These constraints guarantee that the demand of each product s of each customer j on each day t is satisfied by a *One-step direct shipping*, a *Two-steps direct shipping* and/or an *Indirect shipping*:

$$\sum_{i \in I} x_{t-l_{ij}^D}^D + \sum_{i \in I} \sum_{k \in K} x_{t-l_{ikj}^{2D}}^{2D} + \sum_{k \in K} x_{t-l_{kj}^W}^W = d_{tsj} \quad t \in T, s \in S, j \in J$$

3) Vehicle capacity constraints:

These constraints guarantee that the number of vehicles used for the *Warehouse replenishment* of each warehouse from each production plant, for *One-step direct shipping* and for *Two-steps direct shipping* on each day is enough to load on the quantity sent:

$$\sum_{s \in S} x_{tsik}^R = Q y_{tik}^R \quad t \in T, i \in I, k \in K$$

$$\sum_{s \in S} x_{tsij}^D \leq Q y_{tij}^D \quad t \in T, i \in I, j \in J$$

$$\sum_{s \in S} x_{tsikj}^{2DR} + \sum_{s \in S} x_{tsikj}^{2D} = Q y_{tikj}^{2D} \quad t \in T, i \in I, k \in K, j \in J$$

4) *Number of vehicles constraints:*

These constraints guarantee that the total number of vehicles sent on each day from the production plants is not greater than the number of available vehicles:

$$\sum_{i \in I} \sum_{k \in K} y_{tik}^R + \sum_{i \in I} \sum_{j \in J} y_{tij}^D + \sum_{i \in I} \sum_{k \in K} \sum_{j \in J} y_{tikj}^{2D} \leq V \quad t \in T$$

5) *Outsourced transportation cost constraints:*

These constraints allow us to charge the transportation cost corresponding to the number of pallets sent from each warehouse to each customer on each day:

$$\sum_{s \in S} x_{tskj}^W > p z_{tkjp} - 1 \quad t \in T, k \in K, j \in J, p \in P$$

$$\sum_{s \in S} x_{tskj}^W \leq p z_{tkjp} + \sum_{s \in S} d_{tsj} (1 - z_{tkjp}) \quad t \in T, k \in K, j \in J, p \in P$$

$$\sum_{s \in S} x_{tskj}^W \leq \sum_{s \in S} d_{tsj} \sum_{p \in P} z_{tkjp} \quad t \in T, k \in K, j \in J, p \in P$$

$$\sum_{p \in P} z_{tkjp} \leq 1 \quad t \in T, k \in K, j \in J$$

6) *Inventory level constraints at the production plants:*

These constraints define the inventory levels at the production plants:

$$B_{0si} = \bar{B}_{0si} \quad s \in S, i \in I$$

$$B_{tsi} = B_{t-1si} + q_{ts} - \sum_{k \in K} x_{t-1sik}^R - \sum_{j \in J} x_{t-1sij}^D - \sum_{k \in K} \sum_{j \in J} x_{t-1sikj}^{2D} - \sum_{k \in K} \sum_{j \in J} x_{t-1sikj}^{2DR}$$

$$t \in T, s \in S, i \in I$$

$$B_{|T|si} \leq \bar{B}_{|T|si} \quad s \in S, i \in I$$

$$B_{tsi} \geq 0 \quad t \in T, s \in S, i \in I$$

7) *Inventory level constraints at the warehouses:*

These constraints define the inventory levels at the warehouses:

$$I_{0sk} = \bar{I}_{0sk} \quad s \in S, k \in K$$

$$I_{tsk} = I_{t-1sk} + \sum_{i \in I} x_{t-2sik}^R + \sum_{i \in I} \sum_{j \in J} x_{t-2sikj}^{2DR} - \sum_{j \in J} x_{t-1skj}^W \quad t \in T, s \in S, k \in K$$

$$I_{tsk} \geq 0 \quad t \in T, s \in S, k \in K$$

8) *One-step direct shipping constraints:*

These constraints guarantee that *Direct shipping* is not used when it is forbidden:

$$x_{tsij}^D \leq d_{tsj} D_j \quad t \in T, s \in S, i \in I, j \in J$$

9) *Warehouse constraints:*

These constraints guarantee that customer j is not served by warehouse k when it is forbidden:

$$x_{tsikj}^{2D} \leq d_{tsj} W_{kj} \quad t \in T, s \in S, i \in I, k \in K, j \in J$$

$$x_{tskj}^W \leq d_{tsj} W_{kj} \quad t \in T, s \in S, k \in K, j \in J$$

10) *Shipment from production plants constraints:*

These constraints guarantee that no shipment is performed from the production plants on the days in the set Ω :

$$x_{tsik}^R = 0 \quad t \in \Omega, s \in S, i \in I, k \in K$$

$$x_{tsij}^D = 0 \quad t \in \Omega, s \in S, i \in I, j \in J$$

$$x_{tsikj}^{2D} = 0 \quad t \in \Omega, s \in S, i \in I, k \in K, j \in J$$

$$x_{tsikj}^{2DR} = 0 \quad t \in \Omega, s \in S, i \in I, k \in K, j \in J$$

11) *Definition of the decision variables constraints:*

$$x_{tsik}^R, x_{tsij}^D, x_{tsikj}^{2D}, x_{tsikj}^{2DR}, x_{tskj}^W \geq 0 \quad t \in T, s \in S, i \in I, k \in K, j \in J$$

$$y_{tsik}^R, y_{tsij}^D, y_{tsikj}^{2D} \geq 0$$

$$\text{integer} \quad t \in T, s \in S, i \in I, k \in K, j \in J$$

$$z_{tkjp} \in \{0,1\} \quad t \in T, k \in K, j \in J, p \in P$$

Appendix B

Table 2: Average initial and final inventory levels and average demand for each reference

Reference	B_{0si}		$B_{ T si}$	I_{0sk}		d_{sj}
	Plant 1	Plant 2	Plant 1 & Plant 2	War. 1	War. 2	
1	42	0	74	90	12	28
2	6	136	253	235	61	98
3	141	0	181	251	67	71
4	0	67	121	232	63	42
5	0	15	34	69	18	10
6	0	14	28	73	21	9
7	35	0	70	100	26	26
8	105	0	177	178	51	70
9	89	0	180	163	44	68
10	81	0	125	145	40	47
11	56	0	132	146	35	47
12	99	0	177	207	46	71
13	9	0	17	23	7	5
14	22	0	54	108	31	21
15	40	0	103	120	23	38
16	25	-	54	81	19	18
17	3	-	11	21	6	4
18	<1	<1	2	0	0	<1
19	<1	<1	2	0	0	<1