Considering greenhouse gas emissions in a single vendor-multiple buyer coordinated supply chain

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Abstract: Today, environmentally conscious operations have become a necessary condition for a supply chain to be sustainable and profitable. This paper investigates this question quantitatively, with reference to a two-echelon coordinated supply chain that consists of a single vendor and multiple buyers. An optimization model is developed that accounts for greenhouse gases emissions caused by the vendor's production process. According to the European Union Emissions Trading System, emission costs and penalties paid for exceeding emissions' limits are taken into account. The objective is to determine the order quantity of each buyer, the number of shipments from vendor to buyers and the production rate of the vendor that minimize the long-run expected joint total cost per time unit. To approach this problem, a heuristic algorithm is presented. Numerical experiments are finally carried out to analyse the effect of different emissions trading schemes.

Keywords: Supply chain; Greenhouse gas; Inventory; Optimization; JELS

1. Introduction

In recent years, an ever increasing environmental awareness of consumers and governments across the world has been modifying business practices in almost all industry sectors (Garetti & Taisch, 2012). On the one hand, companies are progressively forced to adhere to international standards and regulations, or, in some cases, they commit themselves to comply with voluntarily set environmental performance targets in order to market or enhance their "green-image" (Martí *et al.*, 2015). On the other hand, the new incentives and cost mechanisms (see for instance the European Union Emission Trading System (EU, 2017)), introduced by governments and international agencies to limit the environmental impact of human activities (*i.e.*, greenhouse gas – GHG – emissions), may severely affect the growth and profitability of whatever business if not properly addressed (Bazan *et al.*, 2015).

Performing environmental conscious operations has become the subject of numerous researches in academic and practitioners' literature, and it has given birth to new terms, such as "Green Supply Chains", "Eco-friendly Supply Chains", and "Sustainable Supply Chains" (Gurtu *et al.*, 2015). A central point of Green Supply Chain Management (GSCM) is also that of optimizing the operations decisions in production, transportation, and inventory to reduce the GHG emissions (Hua et al., 2011). The problem of incorporating environmental issues into inventory systems has been receiving a sustained attention in recent years (Bazan *et al.*, 2015). In this regard, a seminal work is owed to Bonney & Jaber (2011) in which they proposed several environmental inventory performance metrics and an "environmental-EOQ" model to account for GHG emissions costs. Later, several works have been published in literature with the aim of integrating the cost of GHG emissions into the supply chain total cost function. Wahab *et al.* (2011) addressed the problem of determining for different scenarios the optimal production-shipment policy for a two-level supply in presence of lots with imperfect quality items. The cost of GHG emissions are included in transportation costs. El Saadany *et al.* (2011) considered a two-level supply chain in which demand is sensitive to price and quality, which are decision variables. Product quality measures environmental issues in an aggregated fashion. Some authors focused on carbon management strategies under the carbon emissions trading mechanism (Hua *et al.*, 2011; Chaabane *et al.*, 2012). Glock *et al.* (2012) addressed the impact of coordination on sustainability in a supply chain consisting of a supplier and a manufacturer. Both the supplier and the manufacturer can control emissions and scrap by varying the respective production rate or by investing in production processes. Jaber *et al.* (2013) optimised the joint production-inventory policy for a vendor-buyer supply chain with GHG emissions cost and penalty cost for exceeding the allowed limits. Benjaafar *et al.* (2013) provided simple but effective optimization models to support decision making in supply chain collaboration, while taking into account the effect of different emission regulations.

It is worth noting that the topic of GHG emissions in a single vendor-multiple buyer supply chain with stochastic demand has been neglected in literature so far. The aim of this paper is to fill this gap.

2. Model development and problem formulation

In this section, we present a cost model that accounts for GHG emissions in a single vendor-multiple buyer coordinated supply chain. The related optimization problem is then formulated.

To establish the mathematical model, the following notation and assumptions are required. Additional notation and hypotheses will be given as needed.

2.1. Notation

Decision variables

Q_n	Order quantity of buyer *n* [quantity]
P	Production rate of the vendor [quantity/time unit]
m	Number of shipments from vendor to buyers

Parameters

D_n	Demand rate of buyer *n* [quantity/time unit]
σ_n	Standard deviation of demand rate of buyer *n* [quantity/time unit]
A_n	Ordering cost of buyer *n* [money/order]
h_n	Unit stockholding cost rate of buyer *n* [money/quantity/time unit]
π_n	Stockout cost per unit shortage of buyer *n*
z_n	Safety factor of buyer *n*
L_n	Replenishment lead time of buyer *n*
q_n	Allowable stockout probability for buyer *n*
S	Setup cost at the vendor [money/setup]
h	Unit stockholding cost rate of the vendor [money/quantity/time unit]
$\overline{P}$	Maximum production rate [quantity/time unit]
$\underline{P}$	Minimum production rate [quantity/time unit]
a	Emissions function parameter [ton·time unit²/quantity³]
b	Emissions function parameter [ton·time unit/quantity²]
c	Emissions function parameter [ton/quantity]
e	Emissions tax [money/ton]
p_i	Emissions penalty for exceeding emissions limit *i* [money/time unit]
E_i	Emissions limit *i* [ton/time unit]
N	Number of buyers
M	Number of emissions limits

Random variables

X_n	Lead-time demand of buyer *n*

Functions and operators

Pr	Probability function
$E[\cdot]$	Mathematical expectation
Ψ	Standard normal loss function
x^+	Maximum between 0 and *x*

Sets

$\mathbb{Z}^+$	Positive integer numbers

2.2. Hypotheses

1. A single vendor supplies one item to multiple buyers.
2. The demand of one buyer is independent of that of the others.
3. Buyer *n* orders a quantity Q_n, such that $Q_n = QD_n / D$ with $Q = \sum_{n=1}^{N} Q_n$ and $D \equiv \sum_{n=1}^{N} D_n$, and the vendor manufactures mQ units with a finite production rate P, such that $\overline{P} \geq P \geq \underline{P} > D$, in one setup. Then, the vendor ships in quantity Q over m times to meet the demand of all buyers.
4. Buyer *n* adopts a continuous review policy and places an order whenever the inventory level falls to the reorder point r_n, which is given by $r_n = D_n L_n + z_n \sigma_n \sqrt{L_n}$, where $D_n L_n$ is the average lead-time demand and $z_n \sigma_n \sqrt{L_n}$ is the safety stock. The safety factor z_n satisfies $\Pr(X_n > r_n) = q_n$.
5. The lead-time demand of buyer *n* is Gaussian with mean $D_n L_n$ and standard deviation $\sigma_n \sqrt{L_n}$.
6. Shortages are allowed and fully backordered.
7. The lead time L_n is deterministic and constant, for each *n*.
8. The maximum production rate $\overline{P}$ is finite because of technical constraints.
9. The time horizon is infinite.

In accordance to Bogaschewsky (1995), the relation between the production rate of a process and the rate of generating greenhouse gas (*i.e.*, CO_2) emissions E can be expressed as follows:

$$E(P) = aP^2 - bP + c. \tag{1}$$

Note that E is measured in [ton/quantity unit]. Equation (1) has been empirically validated for car engines. Moreover, it has been endorsed in literature to model the relation between production rate and greenhouse gas emissions (see, *e.g.*, Jaber *et al.* (2013)).

Given the emissions per quantity unit E obtained by means of Eq. (1), the amount of GHG emitted to satisfy the demand D with a production rate P is $DE(P)$. Hence, the cost for emissions per time unit C_E is given by (Jaber *et al.*, 2013)

$$C_E(P) = eDE(P) + \sum_{i=1}^{M} p_i Y_i(P), \tag{2}$$

where

$$Y_i(P) = \begin{cases} 1 \text{ if } DE(P) > E_i \\ 0 \text{ otherwise.} \end{cases}$$

The first addendum in the expression of C_E represents the tax that must be paid in relation to the overall quantity of GHG that is emitted. The second addendum in the right-hand side of Eq. (2) takes into account the penalties that must be paid if emission limit E_i is exceeded.

2.3. Cost model and related optimization problem

The long-run expected joint total cost per time unit for the considered system consists of the ordering cost and the inventory holding cost for the buyers, the setup cost and the inventory holding cost for the vendor, and the cost for emissions. According to the hypotheses and notation given in the previous section, the costs relevant to the vendor and to the buyers are obtained as described below.

With regard to the *n*th buyer, the expected inventory cycle time is Q_n/D_n time units. The average ordering cost per time unit is $A_n D_n/Q_n$. The average inventory is $Q_n/2 + r_n - D_n L_n$, *i.e.*, $Q_n/2 + z_n \sigma_n \sqrt{L_n}$ (Jha and Shanker, 2013). Hence, the expected stockholding cost per time unit is $h_n\left(Q_n/2 + z_n \sigma_n \sqrt{L_n}\right)$. The expected shortage per cycle is $E\left[\left(X-r\right)^+\right]$, and the expected stockout cost per cycle is therefore $\pi E\left[\left(X-r\right)^+\right]$. Since the number of cycles per time unit is D_n/Q_n, then the expected stockout cost per time unit is $\pi_n D_n E\left[\left(X-r\right)^+\right]/Q_n$. According to our assumptions, $E\left[\left(X-r\right)^+\right] = \sigma_n \sqrt{L_n}\Psi\left(z_n\right)$ (Jha and Shanker, 2013). Hence, the expected stockout cost per time unit is $\pi_n D_n \sigma_n \sqrt{L_n}\Psi\left(z_n\right)/Q_n$.

For what concerns the vendor, the average production cycle length is mQ/D. The average inventory is $Q/2\left[m\left(1-D/P\right)-1+2D/P\right]$ (Jha and Shanker, 2013). Thus, the expected stockholding cost per time unit is $hQ/2\left[m\left(1-D/P\right)-1+2D/P\right]$. The average setup cost per time unit is $SD/\left(mQ\right)$.

The long-run expected joint total cost per time unit C for the system vendor-buyers is given by the sum of all costs for the vendor and for the buyers, plus the cost for emissions. If we recall that the cost for emissions per time unit C_E is formulated according to Eq. (2), then C is expressed as follows:

$$\begin{aligned} C\left(Q,m,P\right) = & \frac{SD}{mQ} \\ & + h\frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \\ & + \sum_{n=1}^{N}\left[\frac{D}{Q}\left(A_n + \pi_n \sigma_n \sqrt{L_n}\Psi\left(z_n\right)\right)\right. \\ & \left. + h_n\left(\frac{Q}{2}\frac{D_n}{D} + z_n \sigma_n \sqrt{L_n}\right)\right] \\ & + eDE\left(P\right) + \sum_{i=1}^{M} p_i Y_i\left(P\right), \end{aligned} \tag{3}$$

where E is given by Eq. (1).

The objective is to determine the order quantity of each buyer, the number of shipments from vendor to buyers and the production rate of the vendor that minimize the long-run expected joint total cost per time unit. This problem can be formalized as follows:

$$\begin{aligned} \text{(P)} \quad & \min_{(Q,m,P)} C\left(Q,m,P\right) \\ \text{s.t.} \quad & Q > 0 \\ & \underline{P} \le P \le \overline{P} \\ & m \in \mathbb{Z}^+. \end{aligned}$$

3. Optimization approach

In this section, we present a solution method that permits to approach problem (P) heuristically. To this aim, it is possible to prove the following proposition, which establishes some useful properties of the cost function:

Proposition 1. *Let* $\hat{Q}$ *be the value of* Q *that satisfies* $\partial C/\partial Q = 0$, *for fixed* $\left(m,P\right)$. *Let* $\tilde{P} \equiv hD^2\left(m-2\right)/\left[hD\left(m-1\right)+\sum_n h_n D_n\right]$, *and* $Y_i = 0$ *for each* i. *C satisfies the following properties:*

i. *For fixed P, assuming* $hD < h_n D_n$, *and relaxing the integrality constraint on m, C is convex in* $\left(Q,m\right)$.

ii. *For fixed* $1 \le m \le 2$ *and with* $Q = \hat{Q}$, *C is convex in P in the interval* $\left]0,+\infty\right[$.

iii. *For fixed* $m > 2$ *and with* $Q = \hat{Q}$, $\partial^2 C/\partial P^2$ *is strictly increasing in* $\left]\tilde{P},+\infty\right[$. *Moreover, there exists a value* $\hat{P}$ *of P, with* $\hat{P} > \tilde{P}$, *such that* $\partial^2 C/\partial P^2 > 0$ *for* $P > \hat{P}$, $\partial^2 C/\partial P^2 < 0$ *for* $P < \hat{P}$, *and* $\partial^2 C/\partial P^2 = 0$ *for* $P = \hat{P}$.

We first observe that $\hat{Q}$ can be determined in closed form:

$$\hat{Q}\left(m,P\right) \equiv \sqrt{\frac{D\left[\frac{S}{m} + \sum_{n=1}^{N}\left(A_n + \pi_n \sigma_n \sqrt{L_n}\Psi\left(z_n\right)\right)\right]}{\frac{1}{2}\left\{h\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+\frac{1}{D}\sum_{n=1}^{N}h_n D_n\right\}}}.$$

From the property at point No. 3 of Proposition 1, it is possible to deduce that, if $\underline{P} \le \hat{P} \le \overline{P}$, then C is convex for $P > \hat{P}$; while it is concave for $P < \hat{P}$. Hence, $\hat{P}$ identifies the inflection point of C.

The value $\hat{P}$ can be found by means of a search procedure along P, for $P > \tilde{P}$ (note that $\tilde{P} < D$). The numerical procedure to find $\hat{P}$ converges surely on $\hat{P}$. In fact, for a fixed $m > 2$, and with $Q = \hat{Q}$ and $Y_i = 0$, for each i, $\partial^2 C/\partial P^2$ is strictly increasing for $P > \tilde{P}$.

The fact that $\partial^2 C / \partial P^2$ is strictly increasing in $\left]\tilde{P}, +\infty\right[$ also implies that $\partial C / \partial P$ (which will be denoted by $C^{(1)}$) is convex in this interval. Hence, $\hat{P}$ clearly identifies the minimum of $C^{(1)}$ in $\left]\tilde{P}, +\infty\right[$.

Depending on the sign of $C^{(1)}\left(\hat{P}\right)$, it is evident that $C^{(1)}$ can admit either two different intersection points with the P-axis ($C^{(1)}\left(\hat{P}\right) < 0$); a single tangent point ($C^{(1)}\left(\hat{P}\right) = 0$); or no intersection or tangent point ($C^{(1)}\left(\hat{P}\right) > 0$). In the case in which $C^{(1)}$ admits two different intersection points with the P-axis, the smallest one and the largest one are respectively denoted by P^- and P^+ (*i.e.*, $P^- < P^+$).

Let P_0^* be the minimum of C in P, for fixed m, and with $Q = \hat{Q}$ and $Y_i = 0$, for each i. For a fixed $m > 2$, ${}^*P_0$ can readily be obtained according to the previous observations. In particular, the following cases should be considered:

1. $\hat{P} \le \underline{P}$:
   a. $C^{(1)}\left(\hat{P}\right) < 0$:
      i. $P^+ \le \underline{P}$: then $P_0^* \equiv \underline{P}$;
      ii. $\underline{P} < P^+ < \overline{P}$: then $P_0^* \equiv P^+$;
      iii. $P^+ \ge \overline{P}$: then $P_0^* \equiv \overline{P}$;
   b. $C^{(1)}\left(\hat{P}\right) \ge 0$: then $P_0^* \equiv \underline{P}$;
2. $\underline{P} < \hat{P} < \overline{P}$:
   a. $C^{(1)}\left(\hat{P}\right) < 0$:
      i. $P^+ \le \overline{P}$: then $P_0^* \equiv \arg\min\left\{C\left(\underline{P}\right), C\left(P^+\right)\right\}$
      ii. $P^+ > \overline{P}$: then $P_0^* \equiv \arg\min\left\{C\left(\underline{P}\right), C\left(\overline{P}\right)\right\}$;
   b. $C^{(1)}\left(\hat{P}\right) \ge 0$: then $P_0^* \equiv \underline{P}$;
3. $\hat{P} \ge \overline{P}$:
   a. $C^{(1)}\left(\hat{P}\right) < 0$:
      i. $P^- \le \underline{P}$: then $P_0^* \equiv \overline{P}$;
      ii. $\underline{P} < P^- < \overline{P}$: then $P_0^* \equiv \arg\min\left\{C\left(\underline{P}\right), C\left(\overline{P}\right)\right\}$;
      iii. $P^- \ge \overline{P}$: then $P_0^* \equiv \underline{P}$;
   b. $C^{(1)}\left(\hat{P}\right) \ge 0$: then $P_0^* \equiv \underline{P}$.

Let us consider the case in which $1 \le m \le 2$, with integer m. We remind the reader that, according to point No. 2 of Proposition 1, C is convex in P, with $Q = \hat{Q}$ and $Y_i = 0$, for each i, in the interval $\left]0, +\infty\right[$.

In the particular case in which $m = 1$, it is possible to note that C diverges at $+\infty$ at the boundary of $\left]0, +\infty\right[$. Hence, C admits a stationary point, which is denoted by P_w. Evidently, P_w can be obtained by solving $\partial C / \partial P = 0$ in P by means of a numerical procedure.

For $m = 2$, we can prove that C admits a stationary point in $\left]0, +\infty\right[$, which will be referred to as P_w. It is easy to find that $P_w = b/(2a)$.

For $1 \le m \le 2$, with integer m, the minimum P_0^* of C in P, with $Q = \hat{Q}$ and $Y_i = 0$, for each i, can be obtained according to the following cases:

1. $P_w \le \underline{P}$: then $P_0^* \equiv \underline{P}$;
2. $\underline{P} < P_w < \overline{P}$: then $P_0^* \equiv P_w$;
3. $P_w \ge \overline{P}$: then $P_0^* \equiv \overline{P}$.

Each emissions limit identifies at most two values of P in the interval $\left[\underline{P}, \overline{P}\right]$ giving that specific emissions limit. That is, with reference to the emissions limit E_i, at most two values of P, say $P_{1,i}$ and $P_{2,i}$, exist in $\left[\underline{P}, \overline{P}\right]$ such that $DE\left(P_{1,i}\right) = DE\left(P_{2,i}\right) = E_i$. This is evident as Eq. (1) is a second-degree equation in P. We will assume $P_{1,i} < P_{2,i}$. Clearly, for each P such that $P < P_{1,i}$ or $P > P_{2,i}$, we have $DE(P) > E_i$.

For a given E_i, $P_{1,i}$ and $P_{2,i}$ can be found by imposing $DE(P) = E_i$, and then solving the equation in P. This readily gives:

$$P_{1,i}, P_{2,i} = \frac{b}{2a} \mp \sqrt{\left(\frac{b}{2a}\right)^2 + \frac{E_i - cD}{aD}}. \tag{4}$$

Let us reorder the emissions limits in increasing order, *i.e.*, $E_{[1]} < E_{[2]} < \ldots < E_{[i]} < \ldots < E_{[M]}$, where

$$E_{[1]} \equiv \min\left\{E_i\right\} \quad \text{and} \quad E_{[i]} \equiv \min\left\{E_j : E_j > E_{[i-1]}\right\}.$$

According to Eq. (4), each $E_{[i]}$ gives two values $P_{1,[i]}$ and $P_{2,[i]}$, with $P_{1,[i]} < P_{2,[i]}$. Note that the relation $E_{[i]} < E_{[j]}$, for $i < j$, implies that $P_{1,[j]} < P_{1,[i]} < P_{2,[i]} < P_{2,[j]}$.

Owing to the above observations, the below algorithm permits us to find a heuristic solution $\boldsymbol{\pi}^*$, with $C^* \equiv C\left(\boldsymbol{\pi}^*\right)$, to problem (P):

Algorithm 1. Heuristic to approach problem (P)

1: **set** $C^* = +\infty$; $Q \leftarrow \hat{Q}$; $m = 1$
2: **set** $i = 1$
3: **calculate** P_0^*
4: **if** ($P_{1,[i]} \le P_0^* \le P_{2,[i]}$) **then**
5: **set** $i^* \leftarrow i$
6: **else**
7: **set** $i \leftarrow i + 1$
8: **go to** line 4
9: **end if**
10: **set** $P^* \leftarrow P_0^*$
11: **if** ($i^* = 1$) **then**
12: **go to** line 38
13: **end if**
14: **for** ($i = 1, 2, ..., i^* - 2$) **do**
15: **if** ($P_{1,[i^*-i]} \ge \underline{P}$) **then**
16: **set** $C_1 \leftarrow \min_P C(Q, m, P)$ in $\left[P_{1,[i^*-i]}, P_{1,[i^*-i-1]}\right]$
17: **else**
18: **if** ($P_{1,[i^*-i-1]} \ge \underline{P}$) **then**
19: **set** $C_1 \leftarrow \min_P C(Q, m, P)$ in $\left[\underline{P}, P_{1,[i^*-i-1]}\right]$
20: **else**
21: **set** $C_1 = +\infty$
22: **end if**
23: **end if**
24: **if** ($P_{2,[i^*-i]} \le \overline{P}$) **then**
25: **set** $C_2 \leftarrow \min_P C(Q, m, P)$ in $\left[P_{2,[i^*-i-1]}, P_{2,[i^*-i]}\right]$
26: **else**
27: **if** ($P_{2,[i^*-i-1]} \le \overline{P}$) **then**
28: **set** $C_2 \leftarrow \min_P C(Q, m, P)$ in $\left[P_{2,[i^*-i-1]}, \overline{P}\right]$
29: **else**
30: **set** $C_2 = +\infty$
31: **end if**
32: **end if**
33: **set** $P^* \leftarrow \arg\min\left\{C_1, C_2, C(Q, m, P^*)\right\}$
34: **end for**
35: **set** $P_m \leftarrow \max\left\{\underline{P}, P_{1,[1]}\right\}$; $P_M \leftarrow \min\left\{\overline{P}, P_{2,[1]}\right\}$
36: **set** $C_0 \leftarrow \min_P C(Q, m, P)$ in $\left[P_m, P_M\right]$
37: **set** $P^* \leftarrow \arg\min\left\{C(Q, m, P^*), C_0\right\}$
38: **if** ($C^* \ge C(Q, m, P^*)$) **then**
39: **set** $\boldsymbol{\pi}^* \leftarrow \left(\hat{Q}(m, P^*), m, P^*\right)$; $C^* \leftarrow C(\boldsymbol{\pi}^*)$
40: **set** $m \leftarrow m + 1$
41: **go to** line 2
42: **else**
43: **stop**
44: **end if**

We observe that the procedure to find C_1 and C_2 is similar to that used to obtain P_0^*, which has been described after Proposition 1.

If the condition $hD < h_n D_n$ is satisfied, then it is easy to observe that the search over m can be done between a minimum value and a maximum value (of m). If we let $\hat{m}$ be the value of m that satisfies the equation $\partial C / \partial m = 0$, with $Q = \hat{Q}$, then we obtain

$$\hat{m}(P) \equiv \sqrt{\frac{S\left[h\left(\frac{2D}{P} - 1\right) + \frac{1}{D}\sum_{n=1}^{N} h_n D_n\right]}{h\left(1 - \frac{D}{P}\right)\sum_{n=1}^{N}\left(A_n + \pi_n \sigma_n \sqrt{L_n}\,\Psi(z_n)\right)}}.$$

Note that $\hat{m}$ is decreasing in P. Hence, the search over m can be restricted in the interval $\left[\max\left\{\lfloor \hat{m}(\overline{P}) \rfloor, 1\right\}, \lceil \hat{m}(\underline{P}) \rceil\right]$.

4. Numerical experiments

In this section, we investigate the impact of some parameters on the optimal solution. In particular, we address our attention to the effect of emissions taxes and penalties, as well as the possibility of buying additional emissions allowances. These tests permit us to put in evidence some managerial insights.

A supply chain with three buyers (*i.e.*, $N = 3$) is considered. Time is expressed in years and the currency is Euro. The relevant parameter values to each buyer are given in Table 1 (for units of measurement, the reader is referred to the notation list).

Table 1: Parameter values for each buyer

n	D_n	σ_n	A_n	h_n	π_n	q_n	L_n
1	100	125	80	17	100	5%	0.08
2	550	83	65	19	110	5%	0.11
3	300	169	95	21	130	5%	0.06

With regard to the vendor, the related parameter values are h = 13 and S = 110. It is assumed $\underline{P} = 1000$ and $\overline{P} = 6000$. The emissions function E is characterized by the following parameter values (Jaber *et al.*, 2013): $a = 3 \times 10^{-7}$; $b = 0.0012$; and $c = 1.4$.

4.1. Test 1: Effect of emissions taxes

In this experiment, we take $p_i = 0$, for each i. Thus, in this circumstance, emissions penalties are not present. Table 2 shows the solution found by Algorithm 1 and the corresponding cost for different values of e.

Table 2: Effect of emissions taxes

e	Q^*	m^*	P^*	C^*
0.01	228	1	6000	7339
0.5	216	1	2660	7735
10	210	1	2054	9632
100	209	1	2006	26740
1000	209	1	2001	197741

We can note that, as e increases, P^* and Q^* becomes smaller, while C^*, evidently, grows. Thus, for larger emissions taxes, the system tends to reduce the lot size and the production rate. Changes in m^* are not apparent, although, theoretically, as the lot size reduces, the number of shipments tends to increase. Note that, for larger values of emissions taxes, P^* becomes closer to the value of P that minimizes emissions, *i.e.*, $P = 2000$.

Let us consider, for example, the case with $e = 10$. When the system operates at the production rate that minimizes emissions, the minimum expected total cost per time unit is 9641, which is larger than the cost of the solution found with Algorithm 1, *i.e.*, 9632. This suggests the importance of jointly optimizing inventory and emissions.

4.2. Test 2: Effect of emissions penalties

In this experiment, we take $e = 0$, which means that emissions taxes are not considered. We introduce the emissions limit $E_1 = 220$. When this limit is overcome, the system pays the penalty p_1. Table 3 shows the solution found by Algorithm 1 and the corresponding cost for different values of p_1.

Table 3: Effect of emissions penalties

p_1	Q^*	m^*	P^*	C^*
10	228	1	6000	7301
100	228	1	6000	7391
500	213	1	2324	7650
2000	213	1	2324	7650

From Table 3, it is possible to observe that, for sufficiently small values of p_1, it is more beneficial to produce at the maximum rate, so as to minimize costs. However, as p_1 increases, it is more convenient to lower the production rate reaching the value 2324. This value coincides with $P_{2,1}$, *i.e.*, the largest value of P which assures that the emissions limit E_1 is not exceeded. Note that the obtained solution does not change for values of p_1 greater than a certain threshold. We can finally deduce that, as P reduces, Q^* becomes smaller as well.

4.3. Test 3: Effect of emissions penalties and taxes

In this test, we investigate the joint effect of emissions penalties and taxes. We consider $E_1 = 220$ and six different combinations between the values of p_1 and e. In Table 4, the results of this analysis are shown.

Table 4: Joint effect of emissions penalties and taxes

e	p_1	Q^*	m^*	P^*	C^*
0.01	100	228	1	6000	7439
	500	213	1	2324	7652
0.5	100	213	1	2324	7760
	500	213	1	2324	7760
10	100	210	1	2054	9632
	500	210	1	2054	9632

The results in Table 4 confirm those found in the previous experiments. For sufficiently small values of e and p_1, the system is allowed to increase the production rate up to the maximum value, although paying emissions penalties and taxes. As emissions penalties and taxes grow, the production rate reduces, as well as the lot size. Although not evident from numerical results, an increment in penalties and taxes also leads to rise the number of shipments (recall the inverse relation between lot size and number of shipments).

The cost C^* clearly becomes higher with increased emissions penalties and taxes. It is also possible to observe that, when e is sufficiently high, the optimal cost becomes independent of the value of p_1.

4.4. Test 4: Effect of emissions allowance trading

This experiment considers the case of multiple penalties imposed at different emissions limits. This condition corresponds to the cap-and-trade system. This system works as follows: companies that exceed their emissions allowances have to pay for additional emissions certificates on the market. Different values of p_i, for $i = 1,2,\ldots,M$, with $p_i < p_{i+1}$, take into account the fact that buying certificates may be related to increased costs, as the demand for certificates grows (Jaber *et al.*, 2013).

In this test, we take into account two emissions limits, *i.e.*, $E_1 = 220$ and $E_2 = 440$. With regard to e, the same values adopted in Section 4.1, in addition to the value $e = 0$, are considered. Table 5 shows the penalties corresponding to each emissions interval. Table 6 gives the results of this analysis.

Table 5: Emissions intervals and penalties

Emissions intervals	Emissions penalties
$DE \leq 220$	0
$220 < DE \leq 440$	50
$DE > 440$	250

Table 6: Effect of emissions allowance trading

e	Q^*	m^*	P^*	C^*
0	228	1	6000	7541
0.01	218	1	2936	7585
0.5	213	1	2324	7760
10	210	1	2054	9632
100	209	1	2006	26740
1000	209	1	2001	197741

A similar pattern in the results to that noted in the previous tests can be observed here. More precisely, for sufficiently small values of emissions taxes, the system tends to balance inventory cost and emissions penalties. The resulting optimal production rate is observed to be larger than both that minimizing emissions only (*i.e.*, 2000) and that minimizing inventory cost only (*i.e.*, 1000 – note that the inventory cost is concave in P and the first derivative in P

is positive, for fixed m and $Q = \hat{Q}$). As emissions taxes grow, the production rate becomes smaller, as well as the lot size. For increasing values of e, the production rate tends to the value of P that minimizes emissions. We argue that this behaviour is because the cost related to emissions taxes becomes predominant with respect to the others.

Note that changes in m are not apparent in this experiment as well. Evidently, the chosen parameter values lead to not sufficiently large values of the lot size. In this regard, we remind the reader the inverse relation between lot size and number of shipments.

For what concerns the optimal cost, this clearly becomes higher as e grows. Note that, for e = 1000, the cost is increased of more than seven times with respect to the case with e = 100. While, for e = 100, the cost is nearly three times the cost for e = 10. Hence, it seems that C^* grows quite fast as e increases.

5. Conclusions

Environmental issues have been topping the agendas of governments, international agencies and enterprises for several years. Also, this renewed environmental consciousness has been promoting new focuses for research activities in order to limit the environmental impact of human activities. The objective of this work was to contribute to this topic considering the case of a two-echelon coordinated supply chain. In particular, a model for a single vendor-multiple buyer supply chain was developed. This model incorporates environmental issues by considering GHG emissions generated by the vendor's production process. Emission costs and penalties paid for exceeding emissions' limits, according to the European Union Emissions Trading System (EU-ETS), are included. The objective was to find the order quantity of each buyer, the number of shipments from vendor to buyers, and the production rate of the vendor that minimize the long-run expected joint total cost per time unit. The optimization problem was addressed by developing a heuristic algorithm that is based on some properties of the cost function. Finally, a sensitivity analysis has been conducted in order to get some insights on the separate and joint effect of the factors emissions taxes and penalties. In addition, the effect of a cap-and-trade system (such as EU-ETS) has been analysed. A possible limitation of this work is that, at present, (*i*) it approaches the problem theoretically and lacks a practical application, and that (*ii*) the model does not consider important aspects such as transportation costs and emissions for transporting goods. Future researches may be devoted to overcome the previous two aspects.

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