Multi objective Genetic Algorithms for
Unequal Area Facility Layout Problems:
A survey

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Abstract: The Unequal Area Facility Layout Problem (UA-FLP) comprises a class of extremely difficult and
widely applicable optimization problems arising in diverse areas and meeting the requirements for real-world
applications. In this paper we present our existing works of the past decade showing the evolutions obtained in
the Genetic Algorithms (GA) used to solve the facility layout problem. GA have recently proven their
effectiveness in finding (sub) optimal solutions to many NP-Hard problems such as UA-FLP. A main issue in
such approach is related to the genetic encoding and to the evolutionary mechanism implemented, which must
allow the efficient exploration of a wide solution space, preserving the feasibility of the solutions and ensuring
the convergence towards the optimum. In addition, in realistic situations where several design issues must be
taken into account, the layout problem falls in the broader framework of multi-objective optimization problems.

Purpose

In this paper, we present a recent survey about layout problems based on the research we have presented in the
last ten years. In Section 1, we introduce the unequal area facility layout problems. Section 2 describes the mu
li-objective genetic algorithm. The portioning structure of the layout and the relative genetic encoding scheme are
reported in section 3. Section 4 shows the four objective functions employed in our approach and the ranking
procedure used. The uncertainty which affects the layout problem in the decision making process is reported in
Section 5. Finally, Section 6 concludes the paper with a short summary of the results obtained and describes the
future developments of our research in this field.

Design/methodology/approach

The placement of the facilities in the plant area, often referred to as “facility layout problem”, is known to have a
significant impact upon manufacturing costs, work in process, lead times and productivity. A good placement of
facilities contributes to the overall efficiency of operations and can reduce up to 50% the total operating
expenses (Tompkins et al., 1996). Layout problems are known to be complex and are generally NP-Hard, as a
consequence, a tremendous amount of research has been carried out in this area during the last decades. A few
surveys have been published to review the different trends and research directions in this area, however, these
surveys are either not recent (Drira et al. 2007).

Originality/value

The unequal area FLP has been an emerging topic in the recent years. A large volume of current research in
unequal area FLPs has been conducted to satisfy both quantitative and qualitative aspects in the layout.
In particular the topic of the Multi Objective optimization problems approached by Genetic Algorithms is
nowadays one of the most promising and investigated research field. Further improvements of the
proposed methodology will include the development of a more comprehensive procedure to approach the
decision process, including the aspects related to the intrinsic uncertainty and referring to the typical
methodologies of the approximate reasoning, such as the fuzzy theory.

Keywords: Facility layout problem, Genetic algorithms, Multi objective optimization

1. Introduction

The efficient design of facilities is an important issues facing any manufacturing industry. Sule (1991) and
Islier (1998) indicated that approximately 30–75% of a product’s cost could be attributed to material handling
and transportation, and such costs can be reduced considerably by an effective layout design. The facility
layout problem (FLP) is an optimization problem concerned with finding an optimum arrangement of a
set of facilities in a given area, subject to some qualitative or quantitative objectives and constraints. In the recent years, several heuristic approaches have been proposed in the literature to find optimal solutions to the FLF, including simulated annealing algorithms, tabu search methods, neural networks and genetic algorithms (GA). A GA is a stochastic search technique based on the concept of the survival of the fittest, emulating the mechanisms of the Darwinian evolution, thus achieving a sub-optimal solution via recursive operations of crossover and mutation (Holland J.H., 1975). Traditional Genetic algorithms for the layout problem typically take into account a single objective function representing the total Material Handling Cost, (i.e., the sum of the rectilinear distances weighted by the flow amounts between the centroids of the facilities, multiplied by a unit cost). This formulation is nowadays overcome as the FLF is considered in a more realistic decision making framework, where several design issues are evaluated by a decision maker (DM) in order to select the most satisfactory configuration. According to this formulation several objectives should be taken into account, as well as some feasibility issues, by means of specific qualitative or quantitative criteria (Tuzkaya, U. and Ertay, T. 2004). These criteria should be ultimately evaluated according to the preferences scheme of the DM. In such cases a fully automated procedure is preferred to select at least a set of best solution candidates, thus allowing the decision maker to evaluate a limited number of alternatives. For such purpose the different objectives are frequently combined into a single one by means of some aggregation procedures such as in the weighted sum method. The drawbacks of these methodologies are well documented in the multi-objective decision theory, as well as the benefits of a “true” multi-objective exploration of the solution space, resulting from a Pareto based approach. Pareto approaches (Goldberg, 1989) involve the evolution of the Pareto front constituted by the fitness of a generic individual corresponding to each optimality criterion considered. It has been recognized the GAs belonging to this class generally outperform the non-Pareto Based approaches (H. Tamaki et al,1996). In this paper, we present a recent survey about layout problems based on the research we have presented in the last ten years. The remainder of this paper is organized as follows. Section 2 describes the multi objective genetic algorithm. The partitioning structure of the layout and the relative genetic encoding scheme are reported in section 3. Section 4 discusses the four objective functions employed in our approach and the ranking procedure used. The uncertainty which affects the layout problem in the decision making process is reported in Section 5. Finally, Section 6 concludes the paper with a short summary of the results obtained and describes the future developments of our research in this field.

2. Multi Objective Genetic Algorithms

Early formulations of the facilities layout problem, based on the general quadratic assignment problem (QAP) can be optimally solved by means of enumerative solution approaches such as branch and bound or implicit enumeration. These approaches are NP-Hard problems and it can be solved in a reasonable time only when the problem size is small, they are therefore practically inapplicable to real life layout problems since the number of solutions increases exponentially with the number facilities to be located. In such situations, random search algorithms are the only practicable alternative, although they may just lead to a near-optimal solution. In its classical formulation the UA-FLP involves the minimization of the total material handling cost, however the needs of the real world of dealing with several design criteria such as the space utilization, flexibility, employee satisfaction and safety emerged already in the early stages of research (Muther, R. 1973). Consequently, to be more realistic, some researchers have considered more than a single objective in their solution approach to the UA-FLP. The presence of multiple objectives in a single optimization problem, however, significantly affects the optimization procedure since, for example, it gives rise not only to a single optimal solution but to a set of optimal solutions (largely known as Pareto-optimal solutions). In the absence of any additional preference, each one of these Pareto-optimal solutions cannot be said to outperform any other. Several optimization methods (including the multi-criteria decision-making methods) suggest converting the multi-objective optimization problem to a single-objective optimization problem thus emphasizing one particular Pareto optimal solution. According to this concept several authors combine the different objectives into a single one for example by means of Analytic Hierarchy Process (AHP) methodology (Harmonosky, C. M., and Tothero, G. K., 1992) or using a linear combination of the different objectives (Chen, C. W. and Sha, D. Y. 2005). Lee et al. (2005) propose a genetic algorithm (GA) for multifloor design considering inner walls and passages, using the weighted method approach to minimize the departmental material handling cost and maximizing closeness rating. A similar approach is proposed by, Ye and Zhou (2007), who developed a hybrid GA-Tabu search (TS) algorithm. Over the past two decades, more advanced researches have led to the formulation of multi-objective evolutionary algorithms (MOEAs) (C. A. Coello and Gary B. 2007; K. Deb 2001; R. O. Day,2005), with the objective to find multiple Pareto-optimal solutions in a single run. In fact, since evolutionary algorithms work with a population of solutions, they can be extended to maintain a diverse set of solutions within the same optimization process. As a consequence in the recent years a number of different GAs were suggested to solve multi-objective optimization problems. These approaches resulted in the development of MOGAs with different structures, namely: MOGA-III (Fonseca C. M. and Fleming P. J.,1993), SPEA2 (E. Zitzler et al.,2001), NSGA-II (K. Deb et al.,2002), NSGA (N. Srinivas and K. Deb, 2005), NPGA (J. Horn et al. 1994), MOMGA (Van
Veldhuizen, D. A. and Lamont, G. B. 2000). The structure of a GA involves the production of an initial population and the achievement of a sub-optimal solution via recursive operations of reproduction, crossover, and mutation. The efficiency of the genetic algorithm mainly depends on the encoding method (in this case it is the space partitioning method), on the ranking procedure and on the fitness function (Beasley et al. 1993).

3.Space Partitioning methods and Genetic Encoding

Two formulations have been proposed for Unequal area facility Layout problems (UA-FLPs), namely, the Flexible Bay Structure (FBS) and the Slicing Structure. Flexible Bay Structure. The FBS formulation (Tate DM, Smith AE. 1995; Aiello et al 2002; Konak et al. 2006), involves the subdivision of the layout area in a pre-established number of columns or bays with different widths and number of departments (fig. 1). The layout is represented by a rectangular area of assigned dimension H (height) and W (width). The width of each bay, that is also the width of all departments within the bay, depends on the sum of department areas in the bay. The height of each department, consequently, is derived knowing its area and width. The generic layout can hence be described by simply specifying the number of bays, the number of departments in each bay and the sequence of departments read from top-left to bottom-right. For example, the layout shown in Fig. 1 is characterized by three bays, [3,2,3] departments per bay and the sequence 1,2,3,4,5,6,7,8.

![Bay structure representation](image)

Chromosome: [1,2,3,4,5,6,7,8,] [3,2,3]

Fig.1 Bay structure representation

By means of such representation, the problem complexity is reduced into determining the departments placement order and the total number of departments each bay will contain. FBS has an advantage in that the bays will become candidates for aisle structures and this facilitates users to transform the model into an actual facility design. However, in the bay structure, the floor is always divided in one direction (vertically or horizontally) into rectangular blocks (bays).

Slicing structure (SS). An alternative decomposition scheme for facility layout design is the slicing structure. A slicing structure results from dividing an initial rectangular layout area either in the horizontal or vertical direction completely from one side to the other (so-called guillotine cut) and recursively going on with the newly generated rectangles (Scholz et al. 2010). A solution is represented by a location matrix $M$, which contains information about the relative locations of the departments on the floor. In our representation (Aiello et al. 2012), in order to obtain a uniform genetic encoding scheme, only quadratic matrices are considered. Consequently, given $N$ departments, the rank ($r$) of the corresponding location matrix is $r = \lceil \sqrt{N} \rceil$, where $\lceil x \rceil$ is the ceiling function that denotes the smallest integer greater than or equal to $x$. The number of elements ($r^2$) in the matrix, is thus greater than or equal to the number of departments. When $r^2$ is strictly greater than $N$, ($r^2-N$) dummy departments with null area are introduced. These dummy departments have null material fluxes from/to other departments and are indexed as zero. In this manner, the relative locations of all departments on the floor are determined by $M$. For example, Figure 2 shows a 3x3 location matrix where there are seven (real) departments and two dummy departments.

![The localization matrix](image)

Fig.2 The localization matrix

The floor is partitioned by a guillotine cut in such a way that the area of each block is equal to the sum of areas of the departments included in the submatrix corresponding to the block. The blocks always have rectangular shapes since only either a vertical or horizontal guillotine cut is used for partitioning. Considering all the possible decompositions, the maximum number of cuts in which the matrix can be divided is calculated by the following equation:

$$T = 2r - 2 \quad (1)$$

In our representation, in order to always obtain a quadratic matrix, at each horizontal cut follows a vertical cut and vice versa. This method is illustrated with an example problem with $N = 10$, $L = 9$ and $W = 10$. Let L and W be the length and the width of the floor, respectively. Figure 3 shows a possible decomposition and its corresponding layout, resulting from the following sequence of cuts: vertical, horizontal, horizontal, vertical and horizontal.

![Slicing structure representation](image)

Chromosome: [8,0,7,0,10,0,0,2,6,3,4,1,5,0,0,9] [1,0,0]

Fig.3 Slicing structure representation

Both representations are affected by a problem of redundancy which means that different chromosomes may result in the same decomposition scheme, or in
other words that individuals with different genotypes may present the same phenotype. The main advantage of the SS representation is its capability to explore a wider solution space, considering that slicing structure includes the FBS when all the cuts are vertical.

4. Objective Functions and Ranking Procedure

Objective Functions. According to the objective functions and the multi-objective approach, our researches (Aiello et al. 2006) have focused on four different aspects of the block layout problem, namely: the handling cost, the adjacency requests, the distance requests and the aspect ratio of departments. The following objectives are therefore considered:

1. Minimization of material handling cost,
2. Maximization of the satisfaction of weighted adjacency,
3. Maximization of the satisfaction of distance requests,
4. Maximization of the satisfaction of aspect ratio requests.

Material handling cost

This objective function expresses the total material handling cost to be minimized:

\[ cost = \sum_i \sum_j (f_{ij} c_{ij}) d_{ij} \] (2)

where \( f_{ij} \) is the material flow between the departments \( i \) and \( j \), \( c_{ij} \) is the unit cost (the cost to move one unit load one distance from department \( i \) to department \( j \)) and \( d_{ij} \) is the distance between the centres of departments using a pre-specified metric.

Closeness request

The second objective has been modeled by the maximization of the adjacency function:

\[ adjacency = \sum_i \sum_j r_{ij} l_{ij} \] (3)

where \( r_{ij} \) is the closeness rating and \( l_{ij} \) is the contact perimeter length between departments \( i \) and \( j \).

Distance request

The third aspect taken into account is the distance request among some departments. The objective function, to be maximized, is expressed in this case by the following equation:

\[ separation = \sum_i \sum_j s_{ij} d_{ij} \] (4)

where \( s_{ij} \) is the distance rating of departments \( i \) and \( j \) and \( d_{ij} \) is the distance between the centres of departments using a pre-specified metric.

Aspect ratio

For each department, a proper aspect ratio is required, typically to optimize the placement of the machines inside. Let \( h \) and \( w \) be the two dimensions of the rectangle; the aspect ratio of department \( j \) is defined as:

\[ Y_j = \frac{\max\{h, w\}}{\min\{h, w\}} \] (5)

Shape ratio is still considered acceptable, with a decreasing level of satisfaction as long as it deviates from the optimal shape, within a certain interval. When the aspect ratio is not acceptable, the score function drastically drops to 0 because manufacturing resources cannot be placed inside the department anymore. The simplest shape of such score function is given in Fig. 4, where \( ar_{ij} \) represents the aspect ratio satisfaction function and \( Y_j^{opt} \) represents the optimal aspect ratio. The presence of even a single department with unfeasible shape makes the whole layout unfeasible. In order to simplify the input data, a unique satisfaction function has been hypothesized for each department and the aspect ratio satisfaction function of each department is shown in Fig. 4. In particular defined in the interval \( \gamma < \gamma^{opt} < 2.5 \) and reaching the optimal aspect ratio in correspondence to \( \gamma = 1.5 \).

 ranking Procedure. A crucial aspect that drastically affects the convergence of a MOGA is the procedure for the selection of best individuals in the population. In a Pareto-Based approach, the ranking procedure is referred to the degree of dominance, which means that, at first the non-dominated individuals in the population are identified, and they are given the rank 1, and removed from the population, then, the non-dominated individuals within the reduced population are identified and given the rank 2, followed by their removal from the population. This procedure is repeated until the whole population is ranked. The least dominated solutions, thus determined, survive to make the population of the next generation. It must be pointed out that individuals belonging to the same front cannot be further differentiated (and ranked) unless an additional elitist mechanism (elitism) is introduced. On one hand this means that the population size must be big enough to involve the whole set of non-dominated solutions, and to maintain the population well differentiated, while on the other hand this suggests the employment of specific operators to maintain a good spread of solutions. The algorithm presented in Aiello et al. 2013
includes a distance-based elitist mechanism to get an estimate of the density of the solutions surrounding a particular solution in the population, according to the referenced crowding distance method (Deb et al., 2002). In particular the average distance of two points on either side of a generic point along each of one the objectives is calculated, and this quantity provides an estimate of the perimeter of the cuboid formed by using the nearest neighbors as the vertices (call this the crowding distance). The crowding-distance computation requires sorting the population according to each objective function value in ascending order of magnitude. Thereafter, for each objective function, the boundary solutions (solutions with smallest and largest function values) are assigned an infinite distance value. All other intermediate solutions are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. This calculation is continued with other objective functions. The crowded-comparison operator guides the selection process towards a uniformly spread-out Pareto optimal front. Finally a clone control routine counts the clones in the population and operates a recursive mutation until the mutated element is different from all the others. This is performed until the number of clones is null or lower than a pre-established acceptance threshold.

5. Uncertainty & Subjectivity in the Decision Making Process

Most of the methodologies proposed in literature for solving plant layout deal with deterministic parameters; however, a deterministic environment, although easier to deal with is not generally the typical environment under which jobshops do operate. For example it could be difficult to determine exactly the amount of the fluxes between the departments, or the area of some departments, or the material handling resources and costs. Some authors approached the layout problem with uncertain parameters by generating a stochastic model, in this situation, the material flows between the departments are uncertain, but an analytical approach involving probability distributions and stochastic variables would be very complex (Shore and Tompkins 1980). A different methodology to approach the uncertainty in the layout problem is the fuzzy theory, which can be formulated in terms of fuzzy sets (Aiello G. & Enea M., 2001; Enea et al., 2005).

Additionally, the decision maker might be uncertain in assigning a precise score to the distance or closeness requests, or to the aspect ratio. To take into account such uncertainty and subjectivity into a transparent and structured methodology, the solutions obtained can be finally ranked by means a multi criteria decision making methodology according to the set of criteria. The search before multi-criteria decision category is generally preferable compared with traditional multi-objective optimization algorithms named multi-criteria decision-making before search, which rely upon the establishment of a normalized weight vector. The major drawbacks of the application of such procedure is that frequently the decision maker is unable to establish proper weights for each objective and that weight-based procedures are compensative, which means that, for a generic solution, a very bad score in one objective may be compensated by good scores in other objectives. For this reason Electre methodology can be implemented as for example in Aiello et al. 2006 and Aiello et al 2012.

6. Conclusions

The development of advanced solution procedures for UA-FLP is a topic frequently undertaken by researchers. In particular, Multi Objective optimization problems approached by Genetic Algorithms is now one of the most promising and investigated research fields. In the most recent formulations, the problem is perceived in association with its multi-criteria structure, paced with up-to-date researches in the field of multi-objective optimization. The research in these fields, has nowadays led extremely complex iterative solution algorithms and optimization procedures. The complexity of such procedures eventually makes it difficult to measure their performance in a fair and reliable benchmark procedure. The analysis of the performance of these algorithms is in fact rarely addressed in the literature, and the lack of a common test bed for benchmarking purposes makes it difficult to comprehend the meaningfulness of the results. In such complex problems, in fact, the effectiveness and reliability of a solution procedure should be tested against a significant set of instances with different dimensions and features which actually does not exist in the literature. This field hence should be better investigated in further researches. Future research topics may also include the development of a more comprehensive procedure to approach the decision process, including aspects related to the intrinsic uncertainty that affects the layout design process. For example the employment of qualitative evaluation scales rather than precise scores might simplify the evaluations of the decision maker, and such qualitative measures might be easily translated into fuzzy sets.


