Spare parts inventory control model for the aeronautical industry

Francesco Costantino*, Giulio Di Gravio, Riccardo Patriarca, Massimo Tronci

*Mechanical and Aerospace Engineering Department, Sapienza University of Rome, via Eudossiana 18, 00184, Rome – Italy (francesco.costantino@uniroma1.it, giulio.digravio@uniroma1.it, riccardopatriarca rp@gmail.com, massimo.tronci@uniroma1.it)

Abstract: Inventory management of spare parts is one of the most critical issues in the aeronautical industry, given the required level of system availability related to the strategic importance and high stocking costs of the components. A failure of any item on an aircraft could determine its unavailability, so spare parts are in stock in order to reduce the unavailability period and provide a prompt substitution. Therefore the allocation of spare parts is subjected to budget constraints and to a target level of availability. Even if a large number of spare parts increases warehousing costs, every single shortage have a greater impact: the adoption of best-in-class inventory management techniques becomes crucial.

On these considerations, the paper presents an innovative model of spare parts allocation developed for an Italian airline with the aim of minimizing back orders and, at the same time, ensuring an availability of 99% depending on the actual flight plan. Starting from the assumptions of VARI-METRIC, the model considers an original configuration of features combining different skills of maintenance centers in a hierarchical multi-echelon, multi-item, multi-indenture structure. It will be solved by an optimization tool with the aim of identifying which spare parts must be stocked in the warehouse at all times and which ones can instead be ordered only when an actual request is raised, with respect to a cost analysis. The process is based on a real case which evaluates the historic yearly demand and an assessment of the times related to repairing, order and shipment for each item, for each site.

Keywords: spare parts allocation, VARI-METRIC, pattern search

1. Introduction

Spare parts inventory management is usually based on data provided by different sources, as for suppliers, that need to be analyzed in accordance to the target of the organization. Commonly used models can follow two different approaches. The item approach tries to define an economic order quantity and period for each item, without considering possible interactions among different items and availability constraints and aiming at reducing inventory and ordering costs while progressively checking the resulting availability (Cavaleri et al., 2008). The system approach tries to define the logistic support to ensure a specified availability. In this case, an availability-cost function is created in order to evaluate inventory costs associated to a required service level. These methodologies present a common starting point in the studies of Sherbrooke on METRIC. This approach is certainly the most appropriate in all that context where time, cost and availability targets can be achieved through different means and have to respect the different features of complex systems (Commonality, service differentiation, multi-transportation modes, multi-echelon structure, and lateral transshipment).

In particular, the aeronautical industry is one of the sectors most characterized by complex systems which require high levels of backups to comply with availability requirements. Furthermore, a multi-echelon structure is a standard requirement: organizations generally control many sites with different targets and competences. Not all the sites can have a warehouse or a maintenance center as any action on critical items (e.g. motors, shafts, sensors) requires certified skills that can be commonly centralized in dedicated structures. Most components have both a strategic and an economic high value and the problem of minimizing inventory costs, assuring at the same time a high availability, is then crucial, considering also the failure rate of the items.

Flight plans, maintenance and testing procedures generates different consumption of components thus requiring models that allow service differentiation. The spare parts allocation problem is widely discussed, as clearly presented by the literature review in Kranenburg (2006):
many detailed mathematical models have been developed in order to explore and provide models of single features but few papers analyze the effects of possible configurations of multiple features.

This paper gives a contribution to this last stream of research, presenting an original combination of features.

2. Literature review

The original METRIC model developed by Sherbrooke has been extended by Muckstadt (1973) to the MOD-METRIC which includes a hierarchical structure for items with two indenture levels in a two-echelon configuration. Slay (1984) developed the VARI-METRIC model assuming that the mean number of items under repair equals its variance, fitting a negative binomial distribution. Later on, Sherbrooke (1986) extended the VARI-METRIC to a version for two-indenture two-echelon systems. As a different approach, Gupta and Albright (1992) modeled a two-echelon multi-indentured repairable item inventory system using Markov chains. METRIC was designed for military applications (Wang et al., 2012; Wang et al., 2013a) where a particular part or component may be demanded at several stations, replenished by one central main base but its evolutions were frequently applied also in the civil aircraft industry (Perlman and Levner, 2010; Chenyu et al., 2012; Lu and Yang, 2012; Sun et al., 2013; Wang et al., 2013b). As stated by van Jaarsveld and Dollevoet (2012), any airline operator needs maintenance shops to achieve short and reliable repair turnaround times (TATs); supply lead times generally exceed the time that operators can wait so that only an accurate dimensioning of the inventory helps to reach required targets.

This research combines some results of the cited studies, modelling and solving a complex systems with a configuration of multiple features. A two-echelon, two-indenture, multi-item model is presented, providing a realistic solution to the spare parts inventory problem of an Italian airline case study where a maintenance network shows differentiation on resources of the repair centers.

The combination of these features, considering also the constraint on budget and the specific availability target related to the actual flight plan, is an original contribution to the specific literature on the system approach of inventory control.

3. The configuration of features

Technical systems are composed of numerous items, which, in most cases, can be further divided into parts and components on indenture levels. Aircrafts belong to this multi-item structure. In this study, two indenture levels (Line Replaceable Unit and Shop Replaceable Unit) are defined, according to the hierarchical structure described in Figure 1. SRUs are the subset of an LRU’s components that can fail and be repaired in the maintenance sites of the system.

Figure 1: Two-indenture structure.

Considering a complex system with a multi-site structure and a different number of aircrafts for each site, the total availability depend on the availability that any single site can provide. When a failure occurs, the correspondent item has to be immediately substituted and the removed item is sent to maintenance to be repaired. Not all the sites are equivalent as a different amount of spare parts is allocated in any warehouse. The structure of the warehouses is multi-echelon, as presented in Figure 2.

Figure 2: Two-echelon structure.

When a failure occurs, if the item is in stock at the Local Warehouse (LW), it is drawn and immediately replaced to restore the aircraft availability. If the item is out of stock, an order is issued to the Central Department (CD) with an \((s-1, s)\) replenishment policy, as the inventory costs of parts are high enough to have an Economic Order Quantity of 1. The failed item is brought to the Maintenance Centre (MC): an inspection process is
required to identify the type of failure, the related repairing activity and to state if the MC is certified to execute the maintenance, according to its level of resources, skills and competences. Otherwise, in case the qualification of the resources are inadequate, the item is sent to the CD where the Maintenance Department (MD) has an opportune level of resources to execute any operation. Referring to a design stage, commonality is not allowed, the transportation mode is unique and lateral transshipment cannot be implemented, because of the strict levels of control mandatory for these situations, activated only during critical missions.

4. Mathematical formulation

The aim of the model is to provide a spare parts allocation of LRUs and SRUs for both LWs and CD: as a failure of any item on an aircraft determines its unavailability, spare parts are in stock in order to reduce the unavailability period and provide a prompt substitution. The allocation of spare parts to the sites has to fit a budget constraint and to reach a target level of availability of the whole complex system. It is assumed that each failure can be treated (in the MCs or in the M) and the repairing times are independent. Considering a model with only one LRU, the variables are listed as follows:

- $i$: item number. LRU: $i = 0$; SRU: $i = 1...I$
- $j$: site number. CD: $j = 0$; LW: $j = 1...J$
- $m_{i,j}$: yearly demand mean value of item $i$ at site $j$
- $q_i$: SRU's conditioned failure rate on the parent LRU
- $s_{i,j}$: level of inventory of item $i$ at site $j$
- $r_{i,j}$: repairing probability of item $i$ at site $j$
- $T_{i,j}$: repairing time of item $i$ at site $j$
- $O_{i,j}$: order and ship time of item $i$ from the CD to the site

Figure 3 presents the demand cycle of each item: when a LRU is required for substitution of an aircraft, a demand of LRU is generated at the LW to replenish the stock. This demand has to be filled by a repaired LRU. Depending on the capability of inspection at the MC, the LRU demand is addressed at the MC or redirected to the MD. The inspection identifies the SRU to substitute and generates a SRU demand. The SRU demand at the MD is always satisfied by repairing the failed SRU while at the MC it can be satisfied only if the MC is certified to execute the repairing process, otherwise the SRU demand is redirected to the MD.

![Figure 3. Demand cycle of items](image)

The total demand $m_{0,0}$ of an SRU at the CD is the sum of two contributions: (1) the demand of SRU deriving from a failure of LRU, inspected at the MCs, not certified to execute the maintenance operations (2) and the demand of SRU deriving from a failure of LRU inspected at the MD (3).

$$m_{i,0} = \sum_{j=1}^{J} m_{i,j} (1 - r_{i,j}) + m_{0,0} q_i$$

$$m_{i,j} = m_{i,0} r_{i,j} q_i$$

$$m_{0,0} = \sum_{j=1}^{J} m_{0,j} (1 - r_{0,j})$$

As any failure of LRU generates an unavailability of the aircraft, any missed request of substituting an item, due to stock-outs, generates a backorder and reduce the total availability of the complex system. Assigning a distribution of probability to this event, the expected backorder of LRU at each LW is the value that has to be calculated to assess and compare different allocations of stock. Given an amount of stock $s$ and a request $x$, the general formulation of the Expected Backorder $EBO(s)$, probability that $x>s$ (4) and the Variance of the Expected Backorder $VBO(s)$ (5) are represented by:

$$EBO(s) = \sum_{x=1}^{\infty} (x - s) Pr(x)$$

$$VBO(s) = \sum_{x=1}^{\infty} (x - s)^2 Pr(x) - [EBO(s)]^2$$
According to the same assumptions on the VARI-METRIC model, the request of the SRU at the CD can be described by a Poisson distribution (independent failure rates and exponential distribution of failures in time) while the demand of LRU at LWs can be described by a negative binomial distribution as the value of the variance is generally higher than the average.

At the CD, the pipeline of any SRU is composed by the items in maintenance and the Expected Backorder. The expected value and the variance of the pipeline (equal in a Poisson distribution) are related only to the repairing time and to the level of spare parts in stock:

\[
E[P_{lja}] = \text{Var}[P_{lja}] = m_{ja} T_{ja} + EBO \left( s_{ja} | m_{ja} T_{ja} \right) \quad (6)
\]

Any LW presents a pipeline for each SRU composed by the items previously requested and arriving from the CD and the Expected Backorder:

\[
E[P_{lja}] = m_{ja} \left( (1 - r_{ja}) o_{ja} + r_{ja} T_{ja} \right) + f_{ja} EBO \left( s_{ja} | m_{ja} T_{ja} \right) \quad (7)
\]

\[
\text{Var}[P_{lja}] = m_{ja} \left( (1 - r_{ja}) o_{ja} + r_{ja} T_{ja} \right) + f_{ja} \left( 1 - f_{ja} \right) EBO \left( s_{ja} | m_{ja} T_{ja} \right) + f_{ja}^2 VBO \left( s_{ja} | m_{ja} T_{ja} \right) \quad (8)
\]

where

\[
f_{ja} = m_{ja} (1 - r_{ja}) / m_{ja} \quad (9)
\]

\[
\sum_{j=1}^{N} f_{ja} = 1 \quad (10)
\]

is the fraction of i-th SRU at the CD to supply the j-th LW. Analogously, the LRU pipeline at the LWs can be described by:

\[
E[P_{lao}] = m_{ao} T_{ao} + \sum_{i=1}^{N} f_{ia} EBO \left( s_{ia} | m_{ia} T_{ia} \right) \quad (11)
\]

\[
\text{Var}[P_{lao}] = m_{ao} \left( 1 - r_{ao} \right) o_{ao} + \sum_{i=1}^{N} f_{ia} \left( 1 - f_{ia} \right) EBO \left( s_{ia} | m_{ia} T_{ia} \right) ^{2} + \sum_{i=1}^{N} f_{ia}^2 VBO \left( s_{ia} | m_{ia} T_{ia} \right) \quad (12)
\]

where

\[
f_{ia} = m_{ao} q_{ia} / m_{ia} \quad (13)
\]

is the fraction of the i-th SRU needed to repair the LRU.

The values representing the LRU pipeline at the LWs are then:

\[
E[P_{lao}] = m_{ao} \left( 1 - r_{ao} \right) o_{ao} + f_{ao} EBO \left( s_{ao} | m_{ao} T_{ao} \right) + \sum_{i=1}^{N} f_{ia} \left( 1 - f_{ia} \right) EBO \left( s_{ia} | m_{ia} T_{ia} \right) \quad (14)
\]

\[
\text{Var}[P_{lao}] = m_{ao} \left( 1 - r_{ao} \right) o_{ao} + f_{ao}^2 VBO \left( s_{ao} | m_{ao} T_{ao} \right) + \sum_{i=1}^{N} f_{ia}^2 \left( 1 - f_{ia} \right) VBO \left( s_{ia} | m_{ia} T_{ia} \right) \quad (15)
\]

where:

\[
f_{ao} = m_{ao} (1 - r_{ao}) / m_{ao} \quad (16)
\]

\[
\sum_{j=1}^{N} f_{ja} = 1 \quad (17)
\]

The total value of the Expected Backorder of LRU can be expressed by:

\[
EBO^{\text{tot}} = \sum_{j=1}^{N} EBO \left( s_{ja} | m_{ja} T_{ja} \right) \quad (18)
\]

5. Optimization problem

To solve the model of a complex system, where any aircraft is made of different LRUs and has a different flight plan, further variables have to be defined:

- \( b \) LRU family number \( b = 1 \ldots H \)
- \( P_{ja} \) price of item \( i \) of LRU \( b \)
- \( n_{j} \) aircraft number at site \( j \) \( n_{j} = 1 \ldots N_{j} \)
- \( M_{j} \) minimum number of required aircraft at site \( j \)

The target function of the problem (A0) is the total percentage availability of the complex system, resulting from the average availability \( A_{j} \) of any site weighted by the minimum number of required aircraft \( M_{j} \):

\[
\frac{\sum_{j=1}^{N} \left( A_{j} M_{j} \right)}{\sum_{j=1}^{N} M_{j}} \quad (19)
\]

Given that any site has a minimum required number of available aircrafts to satisfy its flight plan, a failure on an aircraft reduces the contribution of the site to the total availability only if the number of available aircraft is lower than the minimum required. This can be modeled as a redundant system with \( N_{j} \) parallel elements where at least \( M_{j} \) elements are available:
\[ A_j^* = \sum_{n_j=d_j} \left( \frac{N_j}{n_j} \right) (A_j)^{n_j} (1-A_j)^{N_j-n_j} \]  

Considering that each aircraft needs all the LRU to be available, its availability \( A_j \) at site \( j \) can be modeled as a series system of the availability \( A_{h_{i,j}} \) of the LRUs (depending on their relative EBO \( \text{EBO}_{h_{i,j}} \)):

\[ A_J = \prod_{k=1}^{N} A_{h_{i,j}} \]  
\[ A_{h_{i,j}} = 1 - \frac{\text{EBO}_{h_{i,j}}}{N_j} \]

To be accepted, the allocation must satisfy both the EBO (23) and the availability (24) and the budget constraints (25) and the inventory’s dimension constraints (26):

\[ \text{EBO}_{h_{i,j}} < N_j \]  
\[ A_{h_{i,j}} \geq A_{\text{target}} \]  
\[ C_{h_{i,j}} \geq C_{\text{target}} \]  
\[ s_{h_{i,j}} \leq s_{\text{MAX}} \]  
\[ C_h = \sum_{j=0}^{n} \sum_{h=1}^{j} \sum_{i=0}^{s} P_{i} \]

The extensive review by Nowicki et al., (2012) on the computational efficiency of the different algorithms that solve the problem demonstrates how the complexity of the modelling needs approximate methods. The model, implemented in MatLab, can be solved by a direct search, derivative-free algorithm, the so-called generalized pattern search (GPS) (Audet and Dennis, 2003).

6. Case study

In order to test the model and the optimization method, an aircraft type is selected, considering its flight plan and reliability performances, identifying its most critical LRU (the Landing Gear Nose).

The LRU has four SRUs: the Retract Serve Nose, the Servo Steering, the Wheel and Tire Assembly Nose, the NGL Shock. According to a time series analysis, it is possible to identify the failure rate of every LRU and the conditioned failure of every SRU, as described in Table 1.

The Order and Ship Time \( (O_{ij}) \) is strictly related to the item’s dimensions and the site’s distance from the CD.

<table>
<thead>
<tr>
<th>(h,i)</th>
<th>Item Code</th>
<th>( q_{hi} )</th>
<th>( P_{hi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0)</td>
<td>Landing Gear Nose LRU10</td>
<td>0</td>
<td>85.000</td>
</tr>
<tr>
<td>(1,1)</td>
<td>Retract Serve Nose SRU11</td>
<td>0.5</td>
<td>25.200</td>
</tr>
<tr>
<td>(1,2)</td>
<td>Servo Steering SRU12</td>
<td>0.12</td>
<td>17.800</td>
</tr>
<tr>
<td>(1,3)</td>
<td>Wheel and Tire Assembly Nose SRU13</td>
<td>0.14</td>
<td>7.500</td>
</tr>
<tr>
<td>(1,4)</td>
<td>NGL Shock SRU14</td>
<td>0.24</td>
<td>5.000</td>
</tr>
</tbody>
</table>

The level of \( r_{h_{i,j}} \) generates a little demand of SRUs from MC to MD where all the item has a much shorter repairing time due to the higher level of skills and competences. The allocation procedure has been applied to a complex system which approximates main bases’ structure of an airline in the Italian territory. The model is, indeed, composed of a CD and two LWs, a single family of aircraft model \( (N_{CD}=M_{CD}=22; N_{LW1}=15, M_{LW1}=14; N_{LW2}=10, M_{LW2}=8) \), a standard service of 99% and a total budget of € 5,000,000. The allocation of spare parts must also satisfy the constraints \( s_{h_{i,j}} \leq 10 \) (for the LRUs) and \( s_{h_{i,j}} \leq 15 \) (for the SRUs). Values of each variable at each site are presented in Table 2.

<table>
<thead>
<tr>
<th>(j,h,i)</th>
<th>Site</th>
<th>( m_{bi} )</th>
<th>( r_{bi} )</th>
<th>( T_{bi} ) [years]</th>
<th>( O_{bi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1,0)</td>
<td>CD</td>
<td>0</td>
<td>1</td>
<td>0.20</td>
<td>-</td>
</tr>
<tr>
<td>(0,1,1)</td>
<td>CD</td>
<td>-</td>
<td>1</td>
<td>0.16</td>
<td>-</td>
</tr>
<tr>
<td>(0,1,2)</td>
<td>CD</td>
<td>-</td>
<td>1</td>
<td>0.16</td>
<td>-</td>
</tr>
<tr>
<td>(0,1,3)</td>
<td>CD</td>
<td>-</td>
<td>1</td>
<td>0.16</td>
<td>-</td>
</tr>
<tr>
<td>(0,1,4)</td>
<td>CD</td>
<td>-</td>
<td>1</td>
<td>0.16</td>
<td>-</td>
</tr>
<tr>
<td>(0,1,0)</td>
<td>LW1</td>
<td>0</td>
<td>0.6</td>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td>(0,1,1)</td>
<td>LW1</td>
<td>-</td>
<td>0.8</td>
<td>0.45</td>
<td>0.20</td>
</tr>
<tr>
<td>(0,1,2)</td>
<td>LW1</td>
<td>-</td>
<td>0.7</td>
<td>0.55</td>
<td>0.13</td>
</tr>
<tr>
<td>(0,1,3)</td>
<td>LW1</td>
<td>-</td>
<td>0.7</td>
<td>0.60</td>
<td>0.12</td>
</tr>
<tr>
<td>(0,1,4)</td>
<td>LW1</td>
<td>-</td>
<td>0.9</td>
<td>0.82</td>
<td>0.04</td>
</tr>
<tr>
<td>(0,1,0)</td>
<td>LW2</td>
<td>0</td>
<td>0.7</td>
<td>0.81</td>
<td>0.61</td>
</tr>
<tr>
<td>(0,1,1)</td>
<td>LW2</td>
<td>-</td>
<td>0.85</td>
<td>0.49</td>
<td>0.33</td>
</tr>
<tr>
<td>(0,1,2)</td>
<td>LW2</td>
<td>-</td>
<td>0.75</td>
<td>0.60</td>
<td>0.20</td>
</tr>
<tr>
<td>(0,1,3)</td>
<td>LW2</td>
<td>-</td>
<td>0.75</td>
<td>0.90</td>
<td>0.28</td>
</tr>
<tr>
<td>(0,1,4)</td>
<td>LW2</td>
<td>-</td>
<td>0.95</td>
<td>0.85</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The final allocation of items is presented in Table 3. It respects both the availability constraint (99.31%) and the
budget constraint (€ 2.237.100 of total cost of ownership) and the inventory dimension constraint.

<table>
<thead>
<tr>
<th>(j,h,i)</th>
<th>Item (j,h,i)</th>
<th>CD</th>
<th>LW1</th>
<th>LW2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1,0)</td>
<td>Landing Gear Nose</td>
<td>1</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>(0,1,1)</td>
<td>Retract Serve Nose</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>(0,1,2)</td>
<td>Servo Steering</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(0,1,3)</td>
<td>Wheel and Tire Assembly Nose</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>(0,1,4)</td>
<td>NLG Shock</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

7. Conclusions

In this paper a multi-echelon, multi-item, multi-indenture spare parts allocation method has been analyzed, combining the results of previous works available in literature to describe the features of a complex system. The original combination deals with a network where repair centers have different skills and certifications so that they aren’t always able to execute the maintenance or repair processes. To fit constraints on budget and on a specific operational availability target (related to the flight plan of any site), a model was developed to minimize the system wide expected backorder, using a pattern search method.

The results of a numerical example show the applicability of the model and the evolution of the availability level of the system according to the increase of the cost due to spare parts allocation.

Motivations of this paper are strictly related to the requirements of the aeronautical and naval industry, both civil and military, where the spare parts values are very high and availability cannot be decreased. Producers or buyers of such critical products can use the presented model to calculate the economic effort and the best allocation of inventory while at the moment of the design of the logistic support of a fleet. This is always more important in the definition of contractual issues of global service that includes maintenance.

This research can be extended in several directions. One possible extension is to model the use of lateral transshipment, possibly with partial or complete pooling strategies. Another extension is to analyze the requirements of specific scenarios (i.e. during missions, where it is possible to introduce a weighted availability value considering flight hours) where endurance length creates a risk related to the lead time of spare parts supply from the producer to the central depot and from the central depot to the local warehouses. As a further improvement, the risk of obsolescence of repairable systems can be introduced, since the lifespan of the model is very wide and spare parts in inventory can face obsolescence and cause greater losses.

References


