Advancing Hybrid Production Systems through Theoretical Frameworks and Stochastic Models for Performance Estimation

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Abstract: Contemporary research provides a solid foundation through the development of models and methodologies for analyzing performance indicators in single-machine production systems, with particular emphasis on exponentially distributed processing times. However, the transition to more complex and hybrid production systems, incorporating multiple machines operating in parallel, necessitates a new perspective in analyzing and optimizing their performance. This study fits within this context by proposing a new generation of mathematical and stochastic models for estimating the performance of such hybrid systems. The objective is twofold: on one hand, to provide a theoretical framework for understanding and predicting the behavior of more complex production systems; on the other hand, to offer practical tools for optimal work-in-process (WIP) sizing in relation to throughput objectives. Through the application of these models, this paper aims to facilitate the design of more efficient hybrid production lines capable of dynamically adapting to demand fluctuations and operational contexts, thereby maximizing productivity and responsiveness in the production system.

Keywords: Performance indicators, Hybrid production systems, Mathematical models, Stochastic models, Optimization

1. Introduction

The manufacturing industry is a crucial sector that transforms raw materials into finished products, driving economic growth and innovation (Salatiello et al., 2024). In the context of industrial production, the Hybrid Flow Shop model has attracted significant interest, realistically representing the complexity of modern production environments (Burcin et al., 2021). This paradigm is notable for its flexibility in effectively managing production demand fluctuations. The Hybrid Flow Shop configuration is less studied and documented compared to other setups, such as the Simple Flow Shop, which has received considerable attention in the literature, as evidenced in the work by Branda et al. (2021). However, despite extensive theoretical discussion, current literature reveals a significant gap: the absence of practical, direct application of existing complex mathematical models in real production scenarios. Research tends to focus on reworking and adapting existing models to specific problems, such as the management of Automated Guided Vehicles (AGVs) or other devices in production settings, rather than on the direct application of models in hybrid production lines.

For example, Fu et al. (2021) proposed a multi-objective optimization algorithm that balances global and local searches to minimize completion time and total delay. These experiments demonstrate that the proposed algorithm outperforms traditional approaches. Similarly, Geng et al., (2020) address scheduling problems in hybrid production contexts, considering both machines and workers as critical resources. They introduced the concept of "multi-objective hybrid flow shop scheduling problem with dual resource constraints" (DHFSP), aiming to minimize completion time, total delay, and workload balancing for workers. Additionally, Song (2021) focuses on energy planning issues in production environments, considering both production efficiency and energy savings. He proposes a mixed integer linear programming model to minimize completion time and energy consumption, also introducing a hybrid multi-objective optimization algorithm based on teaching-learning to integrate various techniques to enhance solution effectiveness.

This research aims to bridge the gap by exploring the practical applicability of the Hybrid Flow Shop model without simplifications. The study context assumes that each production stage includes multiple parallel machines and that jobs must pass through multiple stages, as defined by Neufeld et al. (2023). The primary objective is to evaluate the mathematical power of the model and use it to define performance parameters and optimize the value of Work in Progress (WIP) in the presence of multiple parallel machines.

Additionally, the future goal is to develop a highly flexible model that can identify performance indicators for hybrid lines with multiple branches. This involves production lines where various types of jobs are subjected to different production cycles and are not required to pass through all the machines present.

The innovative contribution of this work lies in the formulation of a closed mathematical model that allows for the exact and analytical identification of performance evaluation parameters for a hybrid line. The absence of similar applications in the literature underscores the urgency and relevance of this study, which not only provides a theoretical framework for understanding and predicting the behavior of complex production systems but also offers practical tools for optimal sizing of WIP in relation to throughput goals.

The document is organized as follows: Section 2 provides an overview of the current state of research, highlighting gaps and outlining the context in which our study is positioned. Section 3 describes the proposed methodology, with a particular focus on the practical application of the mathematical model. Section 4 presents the conclusions of our study, discussing the potential impact and implications for future research and industrial practice.

2. Related Works

Academic research has devoted considerable attention to the analysis of the Hybrid Flow Shop, a complex production environment characterized by a hybrid configuration of machinery and processes. However, although numerous studies have focused on solving the scheduling problem in this context, a detailed analysis of the performance metrics of production lines has been largely neglected. This lack of thorough examination is significant, considering that the optimization models used to define optimal schedules incorporate objective functions that include critical parameters, which exert a direct influence on the overall performance of the production line.

It is therefore imperative to conduct a more comprehensive examination of the performance metrics of production lines within the context of the Hybrid Flow Shop. Through an in-depth analysis of these indicators, a more complete understanding of the impact of various factors on overall productivity and operational efficiency can be obtained. This research can prove valuable not only for the optimized design and management of hybrid production lines but also for advancing theory and practice in the fields of operations management and industrial engineering.

Furthermore, greater consideration of production line performance metrics can contribute to better integration between production planning theory and industrial practice. Such integration is crucial to ensure that proposed solutions are not only theoretically sound but also pragmatically useful and applicable in real production contexts. Therefore, deeper reflection on these metrics can foster the development of more sophisticated and robust planning and optimization methodologies capable of addressing the complex and dynamic challenges that characterize the contemporary production environment. Firstly, as highlighted in Neufeld et al. (2023)'s study the most common methods for evaluating and analysing hybrid production lines are represented by heuristic and metaheuristic methods, or a combination of both.

For example, Zhang et al. (2022) addresses a multi-objective scheduling problem aimed at optimizing production time and the total number of sub-batches in a hybrid workflow context. They introduce the concept of "lot streaming" to overlap successive operations and propose a multi-objective programming model to manage this complexity. Due to the computational complexity of the problem, metaheuristic solutions are employed, highlighting the importance of algorithmic parameters in the design of such approaches, and presenting an automatic design methodology for creating multi-objective evolutionary algorithms. The superiority of this approach over other advanced algorithms is empirically demonstrated.

Similarly, Bozorgirad et al. (2013) describes a scheduling problem in a production environment with parallel machines and variable execution times. The objective is to minimize the producer's cost by reducing work-in-process inventory and maximize customer satisfaction by reducing tardiness. A mathematical model is developed, and four tabu search-based algorithms are proposed to find optimal or near-optimal solutions. The effectiveness of these algorithms is compared through statistical analysis, identifying the best approach. Finally, the results of the proposed algorithms are compared with solutions obtained from optimization software to confirm their effectiveness.

Hasani et al., (2020) investigate a flexible flow shop scheduling problem, where there are some unrelated parallel machines with different technology levels in the first stage. Solutions are proposed using a mixed-integer linear programming model to minimize the production time and determine the necessary auxiliary resources. Three approaches are compared: genetic algorithm, simulated annealing, and enhanced particle swarm optimization. The proposed algorithm provides accurate results and can be adapted to practical constraints such as budget and machine efficiency.

Chen et al., (2020) address the hybrid flow shop scheduling problem, common in various manufacturing operations. While most research focuses on improving production efficiency, this study emphasizes reducing energy costs and completion time at the manufacturing-system level due to the increasing concern about carbon emissions and energy savings. The proposed approach involves a multi-objective mixed-integer programming model to minimize both production makespan and electric power consumption. Given the trade-off between these objectives and computational complexity, a genetic algorithm is adopted to efficiently obtain approximate Pareto solutions. Additionally, a multi-objective energy efficiency scheduling algorithm is developed to evaluate the fitness values in the genetic algorithm.

Lei et al. (2017) tackle the hybrid flow shop scheduling problem (HFSP) with assembly operations, aiming to minimize total tardiness, maximum tardiness, and makespan simultaneously. It highlights the importance of tardiness objectives, often overlooked in similar studies. A simple strategy is used to optimize key objectives. Additionally, a novel neighborhood search with global exchange (NSG) method is proposed, incorporating a part-based coding method and global exchange to generate high-quality solutions. Extensive experiments demonstrate the effectiveness of the strategy on key objectives and the competitiveness of NSG for HFSP.

Lei et al., (2020) explore the distributed two-stage hybrid flow shop scheduling problem (DTHFSP) in a multifactory setting, a less-studied area compared to single factory scenarios. It focuses on minimizing makespan and the number of tardy jobs, considering sequence-dependent setup times. To address this problem, the shuffled frog-leaping algorithm with memeplex grouping (MGSFLA) is proposed. The algorithm involves an initial population generation using a heuristic, followed by two sequential phases with a new population division. In the second phase, memeplexes are categorized into groups, each undergoing different search processes, and the best memeplex is excluded from further division. Computational experiments on various instances demonstrate the effectiveness of the main strategies and the promising advantages of using MGSFLA.

Schulz et al., (2020) examine the planning problem in a hybrid production environment, considering both energy costs and total delivery delay. They explore the trade-off between lower production speeds to save energy and the need for shorter processing times to ensure on-time delivery. They propose two new models that include energy prices based on usage time and analyse the impact of different production speed levels on energy, energy consumption, and on-time delivery through a numerical case study.

Kong et al., (2020) address the issue of sustainable planning in a hybrid workflow environment, where the selection of parallel machines for various jobs significantly impacts overall sustainability. They propose a new model that considers the characteristics of parallel machines through evaluations of power, production efficiency, and cost. To solve this model, they develop an improved genetic algorithm that accelerates convergence by introducing a matching distance between parallel machines and optimizing objective weights. Finally, they conduct a case study to confirm the effectiveness of the proposed model and algorithm, providing insights for sustainable production through different scheduling modes.

Marichelvam et al., (2020) deal with the planning of multi-phase hybrid workflows, considering the impact of human factors such as different levels of work and learning and forgetting. They propose to minimize a weighted combination of makespan and total flow time. To solve this complex problem, they propose an improved version of the Particle Swarm Optimization (PSO) algorithm, integrated with a dispatching rule and a constructive heuristic. Additionally, they combine the Variable Neighbourhood Search (VNS) algorithm with PSO to obtain optimal solutions in less computation time. They validate the performance of the proposed algorithm on various instances of random benchmark problems.

Finally, Geng et al., (2020) focus on the problem of hybrid workflow planning considering both machines and workers as resource constraints. They propose a multi-objective model to minimize maximum completion time, total delay, and worker load balancing. They introduce an improved multi-objective memetic algorithm (IMOMA), which outperforms other known algorithms in terms of convergence, diversity, and dominance of non-dominated solutions. Numerical experiments confirm the effectiveness of IMOMA in solving the hybrid workflow planning problem with dual resource constraints.

Meng et al., (2020) tackle the problem of flexible job shop distributed planning (DFJSP) with the aim of minimizing maximum completion time (makespan). They propose four mixed integer linear programming (MILP) models and a constraint programming (CP) model, formulated with various modelling ideas. MILP models are effective for small-sized problems, while to address the NP-hard complexity of DFJSP, they propose an efficient CP model. Numerical experiments demonstrate the effectiveness of the proposed models in solving benchmark problems.

Shao et al., (2021) address the problem of efficiently scheduling production resources among decentralized production centers through the study of a multi-objective distributed hybrid flow shop scheduling problem (MDHFSP). The objective is to minimize the total weighted makespan, earliness, tardiness, and workload. A multi-objective evolutionary algorithm based on multiple neighborhoods local search (MOEA-LS) is proposed to solve the MDHFSP. The algorithm uses a weighting mechanism during the initialization phase to determine the best position for each job. Neighborhood search operators based on the three objectives are designed to generate improved solutions. Additionally, adaptive weight updating mechanisms and simulated annealing are utilized to avoid local optima. Results show that the proposed algorithm is highly efficient in solving the MDHFSP, outperforming other classical multi-objective optimization algorithms.

Ying et al. (2018) focus on the distributed hybrid flowshop scheduling problem with multiprocessor tasks, addressing this challenge for the first time. To solve this complex problem, a SIG algorithm is proposed, which incorporates a mixed integer linear programming formulation and an adaptive decoding mechanism. Results demonstrate that the proposed algorithm is highly efficient and effective, thus expanding the research area of distributed scheduling problems. It should be emphasized that heuristic and metaheuristic methods, while widely used to solve mathematically and computationally complex problems, have significant limitations. These approaches, while providing approximate solutions, do not guarantee the search for the optimal solution, compromising the accuracy and precision of the resulting analyses and forecasts. In the context of this research, a significant gap in the literature is highlighted: no author has applied the Buzacott model in its original and complete form, likely due to its numerical complexity and the restrictions imposed by the limits of probabilistic calculation and the need to hypothesize exponential distributions or simulate similar behaviors to model the system with Markov chains. This gap offers a unique opportunity for this study to explore the direct applicability of the Buzacott model in the Hybrid Flow

Shop, overcoming existing challenges and significantly contributing to the existing literature.

In Table 1, a summary table of the state of the art is presented.

Table 1: Overview of the State of Art

Authors	Objective Function	Technique
Bozorgirard & Logendran	minimize cost	GA-GA
Hasani & Hosseini	minimize cost	GA
Chen et al	minimize makespan	NSGA
Lei & Zheng	minimize makespan and total tardiness	NSG
Lei et al	minimize total tardiness	TTBLO
Lei et al	minimize cost	TICA
Zhang et al	minimize cost	GA
Zhang et al	minimize cost	GA
Kong et al	energy efficiency	GA
Marichelvam et al	minimize makespan	PSO
Geng et al	minimize makespan	IMOMA
Meng et al	minimize makespan	CP
Shao et al	minimize makespan	MOEA
Ying et al	minimize makespan	GrA

3. Proposal Definition

This study aims to develop and compare a rigorous mathematical model with a simulation model to analyze the performance parameters of a hybrid production line characterized by parallel machines. Utilizing the EMVA and EMDA algorithms described in "Stochastic Models of Manufacturing Systems" by Buzacott and Shanthikumar (1993), the mathematical model is designed to provide precise analytical benchmarks to evaluate the simulation results. The simulation model, implemented in the AnyLogic environment through a multi-method approach of Discrete Event Simulation (DES) and Agent-Based (AB), produces useful results but lacks a direct analytical reference for comparison.

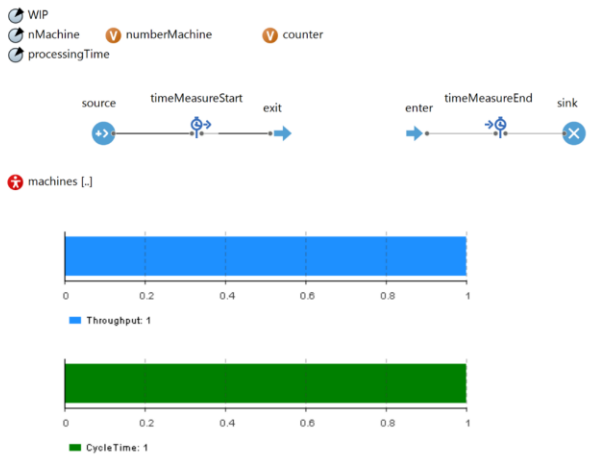

Figure 1. Main Agent

The operational model in AnyLogic leverages two main agents:

- Main: Manages workflows and coordinates interactions among various system components (Figure 1).
- Machine: Simulates the operations of each machine through Queue and Delay blocks, based on processing times defined by the ProcessingTime parameter (Figure 2).

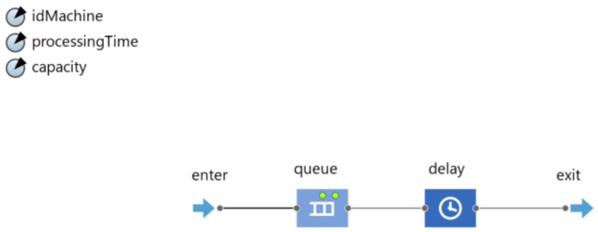

Figure 2. Machine Agent

This model is parameterized on the parameters nMachine, ProcessingTime, and WIP, where nMachine and ProcessingTime are arrays that respectively define the number of machines per Service Center and the processing time per machine. It is worth noting that the simulation model thus conceived enjoys a high degree of generalization and flexibility. The simple manipulation of parameters allows the model to be adapted to various production scenarios. This flexibility represents a significant advantage as it enables the effective exploration and evaluation of various production configurations. System performance, in terms of throughput and cycle time, is visualized through bar charts, providing an immediate and effective analysis of system efficiency.

To test the proposed model, experiments were conducted assuming a number of machines equal to 5 and a number of jobs equal to 10. With these assumptions, the following results were obtained in terms of Throughput and Cycle Time, as shown in Figure 3.

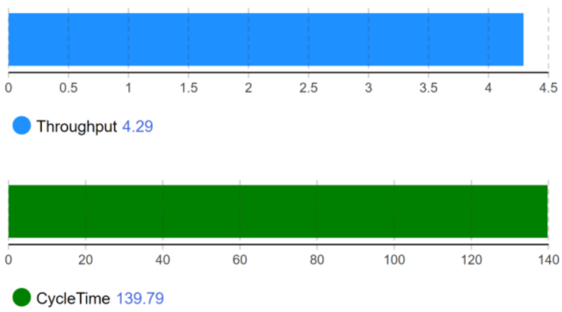

Figure 3. Throughput and Cycle Time bar charts

Key Elements and Variables of the Model:

- Parallel Machines: Analysis of the behavior of stations with multiple machines operating simultaneously.
- Jobs in Transit: Study of workflow through the production line.
- Work Stations: Analysis of the number and function of stations in the line.

Model Inputs and Outputs:

- Input parameters: arrival and service times (i.e. static values or probability), station configuration (number of station and number of machines at each station), and machine parameters (number of machines in parallel and service time).
- Output parameters: Throughput (defined as Job per hour), Cycle Time (defined as the time elapsed between the entry and exit of a Job from the production line).

The EMVA and EMDA algorithms are implemented using Python to analyze the hybrid production line. We have not modified these algorithms; rather, we are applying them in their original form to evaluate their effectiveness in our specific context:

• EMVA (Extended Mean Value Analysis): Although it is a simplified case and widely discussed in the literature, our focus is less on this algorithm since its use and modifications are well documented; it is used to define average queue length, average waiting time, and throughput, as shown in **Algorithm 1.**

Algorithm 1 Extended Mean Value Analysis (EMVA)

Step 1: Set $E[N_i(0)] = 0$, $i = 0, \dots, m$; $TH(0) = 0$.

Step 2: For $l = 1, \dots, n$, compute:

$$E[T_i(l)] = \{E[N_i(l-1)] - v_i \cdot TH(l-1) \cdot E[S_i] + 1\} + v_i \cdot TH(l-1) \cdot E[S_i^2]/2$$

$$TH(l) = \frac{l}{\sum_{i=0}^{m} v_i \cdot E[T_i(l)]}$$

$$E[N_i(l)] = v_i \cdot TH(l) \cdot E[T_i(l)], \quad i = 0, \dots, m$$

Step 3: Stop.

• EMDA (Extended Marginal Distribution Analysis): This algorithm is not widely covered in the literature, making it of particular interest for our study. EMDA allows for detailed analysis of systems with multiple parallel machines, and its implementation poses significant challenges, especially in defining marginal and stationary probabilities, which are crucial for our model. The phases of algorithm are described in **Algorithm 2.**

Algorithm 2 Extendend Marginal Distribution Analysis (EMDA)

Step 1: Set $l = 1$, $r_i^{(l)}(k_i) = \min\{k_i, c_i\}$, $k_i = 1, \dots, n$; $r_i^{(l)}(0) = 0$, $i = 0, \dots, m$ e $\mu_i = 1/E[S_i]$, $i = 0, \dots, m$.

Step 2: Apply the MDA algorithm with $r_i = r_i^{(l)}$, $i = 0, \dots, m$; and obtain

$$h_{i:k_i}(\mu_j r_j, j = 0, \dots, m) = p_i(k_i, n), \quad k_i = 0, \dots, n; \quad i = 0, \dots, m.$$

Step 3: Compute

$$\lambda_i(k_i - 1) = \frac{\mu_i \cdot r_i(k_i) \cdot h_{i:k_i}(\mu_j \cdot r_j, j = 0, \dots, m)}{h_{i:k_i-1}(\mu_j \cdot r_j, j = 0, \dots, m)},$$

$$k_i = 1, \dots, n; \quad \lambda_i(n) = 0; \quad i = 0, \dots, m.$$

Obtain the stationary probability distribution $\{g_{i:k_i}(\lambda_i, G_i, c_i), k_i = 0, \dots, n\}$ of the number of parts in a $M_i(n)/G_i/c_i$ queueing system with state-dependent arrival rate $\lambda_i(k_i)$, $k_i = 0, \dots, n$.

Step 4: If the sum of the absolute differences

$$\sum_{i=0}^{m} \sum_{k_i=0}^{n} |p_i(k_i, n) - g_{i:k_i}(\lambda_i, G_i, c_i)| \le \epsilon,$$

an acceptable error, go to Step 5. Otherwise, set $l = l + 1$, $r_i^{(l)}(k_i) = \min\{k_i, c_i\}$, $k_i = 1, \dots, n$; $r_i^{(l)}(0) = 0$, $i = 0, \dots, m$ and $\mu_i = 1/E[S_i]$, $i = 0, \dots, m$, and go to Step 2.

Step 5: The marginal probability distribution $p_i(\cdot : n)$, $i = 0, \dots, m$, and the throughput rate $TH(n)$ obtained by the MDA algorithm during the last iteration are the approximate performance measures of the general closed queueing network delivered by the EMDA algorithm.

The mathematical model not only aims to accurately assess system performance but also to improve the simulation model. By using the mathematical model results as a benchmark, further optimization and refinement of the simulation model are possible, ensuring greater accuracy and reliability in the results obtained. This integrated approach, combining the advantages of simulation with the precision of the mathematical model, will allow a deeper understanding of the production system dynamics, providing a solid foundation for potential improvements and future optimizations.

At the current stage of the study, the results obtained are comparable, though not identical, between simulation and the mathematical model. Below is a table summarizing the preliminary results obtained (Table 2).

	Throughput [Job/h]	Cycle Time [min]
Simulation	4.29	139.79
Mathematical Model	5.31	112.96

Table 2. Comparison of Result

One of the main challenges is adapting the probabilistic definitions (Rozhok et al., 2019) needed to effectively implement the EMDA. These challenges are being addressed through an iterative approach of calibration and validation, continually comparing the mathematical model results with those from the simulation in AnyLogic. This comparison and validation process will validate the effectiveness of the EMVA and EMDA algorithms in the specific context of a hybrid production line and identify improvement areas for both models.

The primary objective of this study is to achieve an effective and methodologically rigorous comparison between the results obtained through the mathematical model and those derived from simulation, ensuring that both models are based on homogeneous premises and thus directly comparable. Currently, the development of the study is in the phase of defining the necessary adjustments to ensure that the two approaches are dual and comparable in every relevant aspect. It is hoped that clear guidelines can be established to facilitate this comparison, with the further goal of extending the research to broader and more generalizable scenarios. Such extension is more straightforward in the context of simulation, given the parameterized nature of the model used; however, it presents greater complexities in the mathematical approach.

Additionally, the aim is to define a model that can effectively represent a production line characterized by parallel machines and multiple branches, diversifying production cycles based on the various types of products handled.

4. Conclusions

Research in the field of hybrid workflow planning has shown a growing interest in understanding and optimizing the performance of hybrid production systems. These systems, characterized by the presence of parallel machines, pose a complex challenge but also offer significant flexibility in managing production demand fluctuations.

Analytical approaches and metaheuristic solutions have significantly contributed to solving complex scheduling and energy planning problems in hybrid production contexts. However, while such approaches offer approximate solutions, research continues to seek more accurate models and more efficient computational methodologies.

The integration of exact analytical models and simulations has proven to be a promising strategy for gaining a comprehensive understanding of the dynamics of hybrid production systems. A rigorous mathematical model can provide a reference point for evaluating system performance, while simulation allows for rapid exploration of different configurations and production scenarios.

Furthermore, research is increasingly focusing on identifying intuitive mathematical methods that can simplify the computational complexity associated with hybrid production models, enabling more efficient and pragmatic evaluation of system performance.

In conclusion, the evolution of research in the field is moving towards a combination of more accurate models and more efficient computational methodologies, aimed at addressing the analytical and planning challenges present in hybrid production systems and providing a solid foundation for future improvements and optimizations.

However, the current limitations of this study mainly concern the definition of descriptive states of the production line and the resolution of transfer matrices, as well as the determination of stationary probabilities within the intricate and articulated context of hybrid production lines.

Future prospects and developments related to this study are focused on the unique and analytical definition of parameters involved in the analysed laws, with the aim of creating an extremely generalized model. Such a model should not only address the specific issue of a Hybrid Flow Shop, characterized by multiple parallel machines but also provide insights into broader challenges encountered in the literature, such as integrating elements like Automated Guided Vehicles (AGVs) into the context of industrial logistics.

Consequently, further development lies in identifying, within the literature, problems susceptible to being schematized as hybrid systems, in order to formulate a comprehensive analytical framework applicable to diversified hybrid production contexts.

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