Performance Maps for Distribution Network Configuration: Multidimensional Analysis with the Buckingham Theorem

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Abstract: Optimizing Distribution Networks (DNs) is vital for retailers' profitability, impacting supply chain performance in terms of service levels and costs. A key decision in DN configuration is the stock deployment policy, which involves choosing between centralized, decentralized, and hybrid DNs for each Stock Keeping Unit (SKU). This decision is challenging since many variables influence the choice of the optimal (cost-effective) stock deployment policy, and they must be considered simultaneously (e.g., number of customers served, number of distribution centers, SKU unitary cost, SKU backorder cost, etc.). Moreover, retailers can manage thousands of SKUs, therefore the decision on the optimal stock deployment policy must be repeated several times. To simplify this decision, retailers seek support tools that guide in associating SKUs with optimal deployment policies. To address this need, Dimensional Analysis (DA) and, particularly, the Buckingham Theorem (BT) offer promising methodologies. Indeed, after modeling the DN configuration problem in a mathematical form, BT checks the meaningfulness of its governing equations, identifies influential variables, extracts knowledge on how they mutually when influencing the optimal stock deployment policies, and facilitates informed decisions about the option to select. Accordingly, BT allows for comparing different DN configurations, creating performance maps which suggest similar stock deployment decisions for similar (scaled) SKUs, suppliers, distributors, etc. Despite the potential usefulness of these maps, no study has explored the capabilities of BT to address stock deployment decisions. This paper addresses this gap by leveraging BT to develop supporting maps for multidimensional scaling, similarity analysis, and economic performance prediction of centralized, decentralized, and hybrid DNs. The achieved maps will constitute the main results of this study, providing retailers with decision support tools for associating similar DNs with optimal stock deployment policies. These maps offer a visual aid for retailers to make informed decisions on DN configuration, ultimately enhancing supply chain performance.

Keywords: Distribution network configuration; supply chain management; supply chain design; Pi-theorem.

1. Introduction

In their simplest form, Distribution Networks (DNs) consist of two-echelon supply chains, in which one or more suppliers replenish stocks in a set of Distribution Centres (DCs), and then retailers deliver stocks from DCs to meet customers' demand at specific consumption points (Tapia-Ubeda et al., 2020). To ensure efficient retailers' performance and competitive advantage, a crucial task is to optimally configure DNs, which allows for ensuring high service levels while reducing total logistic costs. Configuring DNs involves making various decisions, such as determining the type, size, number, and location of DCs where stocks are temporarily stored on their way to end customers (Alemany et al., 2021). Among the crucial decisions influencing DN configuration, selecting stock deployment policies has been reported as of primary importance, involving a choice between centralized, decentralized, and hybrid DNs (Gregersen and Hansen, 2018). In centralized DNs, all stocks are allocated in a single DC, which serves all customers. The benefits of centralization involve the risk-pooling effect that helps mitigate demand uncertainty, leading to reduced inventory levels, low replenishment orders, and reduced inventory costs. However, it comes at the expense of longer distribution times and decreased DN flexibility. Conversely, in decentralization, stocks are stored in multiple peripheral DCs, each serving local customers. Decentralization offers advantages such as high DN flexibility due to shorter distances between DCs and customers, resulting in quicker distribution times and high service levels. Nevertheless, it comes with increased inventory costs, higher replenishment orders, lower inventory turnover, and no risk-pooling. Finally, in hybrid DNs, trade-off benefits are achieved since an intermediate number of DCs is selected between one (centralization) and one per local customer (decentralization). When evaluating the choice between centralized, decentralized, and hybrid DNs, three main challenges emerge (Cantini et al., 2022a). Firstly, retailers typically handle thousands of

Stock Keeping Units (SKUs), requiring a tailored stock deployment policy for each of them (depending on individual factors such as product type and demand) (Mangiaracina et al., 2015). Secondly, numerous influential variables affect the decision for the optimal stock deployment policy (e.g., the number of customers, DC location with respect to customers, logistic costs, etc.). These variables must be considered simultaneously due to their mutual interactions affecting DN characteristics (Cantini et al., 2023). Lastly, in analysing different combinations of influential variables and their effect on DN configuration performances, the optimal solution must be selected by finding a trade-off between conflicting needs, such as increasing service levels while reducing logistics costs. Due to these three challenges and the multitude of stock deployment policies (i.e., centralized, decentralized, and hybrid DNs), configuring DNs is a complex task that necessitates the adoption of structured methodologies. Among existing methodologies, the majority involve solving mathematical programming models to maximize the performance of specific DNs (Biuki et al., 2020). Conversely, according to (Cantini et al., 2022b), few studies propose direct comparisons between the performance of different stock deployment policies. Consequently, businesses lack overarching guidelines to assess broader scenarios and discern the advantages of adopting centralized, decentralized, or hybrid DNs.

Against this backdrop, Dimensional Analysis (DA) emerges as a valuable method to assist retailers in DN configuration. Indeed, by converting a general problem (e.g., the DN configuration) into a mathematical form, DA examines the meaningfulness of its governing equations. Therefore, DA identifies influential variables and how they interact when affecting outcomes. Consequently, DA allows retailers to predict the performance of centralized, decentralized, and hybrid DNs, determining the optimal (i.e., most cost-effective) alternative. DA achieves its highest fulfilment with the Buckingham Theorem (BT) (Miragliotta, 2011). BT states that any set of equations describing a problem can be simplified using dimensionless variables known as 'dimensionless groups'. Specifically, the number of influential variables describing a problem can be decreased by the number of independent physical dimensions (length, mass, time, etc.) present in its modelling equations. As a result, an original problem $F(A_1, A_2, \dots, A_n) = 0$ characterized by n (dimensional) influential variables described by k fundamental dimensions, can be simplified to a function of $(n-k)$ dimensionless groups, thus becoming $\Phi(\pi_1, \pi_2, \dots, \pi_{n-k}) = 0$ (Brunetkin et al., 2018). Due to this statement, BT promises to assist retailers in DN configuration in three ways. First, by identifying a reduced set of dimensionless groups that govern the problem's outcomes, BT diminishes the number of variables necessary to evaluate when predicting the performance of different stock deployment policies. Second, BT enhances understanding of the DN configuration problem, highlighting relationships among its influential variables. Lastly, BT enables similitude and scaling considerations. Particularly, according to BT, DN configurations sharing identical values of dimensionless groups (despite differing influential variables) can be generalized to achieve the same performance. Hence, as traditionally done in other sectors such as turbomachinery (Bicchi et al., 2023), BT facilitates the creation of performance maps for DN configurations, where the performance of centralized, decentralized, and hybrid scaled DNs is shown. By referring to scaled DNs, companies can put themselves in similarity and make strategic choices of DN configuration.

Despite the potential usefulness for retailers, the application of BT in the field of DN configuration (and more broadly, in Operations Management) has been overlooked by the literature (Miragliotta, 2011). This paper aims to bridge this gap by exploring, for the first time, the use of BT to achieve a generalized dimensionless model for comparing the economic performance of centralized, decentralized, and hybrid DNs. As the outcome, ten dimensionless groups are identified to predict the performance of scaled DN configurations. Therefore, retailers are provided for the first time with visual tools (i.e., performance maps) through which they can compare scaled DNs, and select the most cost-effective stock deployment policy for their company based on similitude analyses. This paper is organized as follows. Section 2 details the mathematical equations considered to express the DN configuration problem. Section 3 describes how to apply BT in DN configuration. Section 4 shows the results of BT application in DN configuration, discussing how retailers can generate performance maps and showing their use in an example case study. Finally, Section 5 offers some conclusions.

2. DN configuration problem

To determine the optimal stock deployment policy for each SKU, the mathematical model by (Cantini et al., 2022b) was adopted, which relies on the notation in Appendix A. Three main stock deployment policies were compared (i=1, 2, and 3 in Figure 1), which differ based on their degree of centralization (Deg_i, Equation 1). In Equation 1, DC_i is the number of DCs able to fulfil the customer demand while N is the number of customers served. Deg_i ranges between 0 and 1, where 0 is decentralization, 1 is centralization, and 0.50 is a hybrid DN configuration.

$$Deg_i = \begin{cases} 1, if\ i = 5\ (centralization) \\ 1 - \frac{DC_i}{N}, else \end{cases} \tag{1}$$

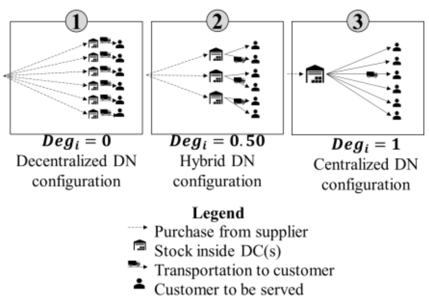

Figure 1. Stock deployment policies investigated

For brevity, here we considered only three main stock deployment policies but the work could easily be extended to any other alternative with Deg_i between 0 and 1. Among the three stock deployment policies (i), the mathematical model selects the optimal one by seeking the alternative with the minimum total cost (C_{TOT_i}, Equations 2-3), which also respects a pre-established service level (SL). C_{TOT_i} is the sum of several cost items, which are detailed in Table A1 and calculated with Equations 4-16.

$$min[C_{TOT_i}]\ for\ i = 1,2,\dots,5 \quad (2)$$
$$C_{TOT_i} = C_{P_i} + C_{H_i} + C_{O_i} + C_{B_i} + C_{T_i} \quad (3)$$
$$C_{P_i} = c \cdot D_i \cdot DC_i \quad (4)$$
$$C_{H_i} = h \cdot c \cdot I_i \cdot DC_i \quad (5)$$
$$C_{O_i} = o \cdot N_{o_i} \cdot DC_i \quad (6)$$
$$C_{B_i} = b \cdot N_{b_i} \cdot DC_i \quad (7)$$
$$C_{T_i} = t \cdot N_v \cdot N_{t_i} \cdot d_i \cdot DC_i \quad (8)$$
$$D_i = \frac{\bar{D} \cdot N}{DC_i} \quad (9)$$
$$DC_i = \begin{cases} 1, if\ i = 5 \\ [(1 - Deg_i) \cdot N]^+,\ else \end{cases} \quad (10)$$
$$I_i = \left(\frac{Q_i}{2} + SS_i\right) \quad (11)$$
$$N_{o_i} = \frac{D_i}{Q_i} \quad (12)$$
$$N_{b_i} = [(1 - SL) \cdot D_i]^+ \quad (13)$$
$$N_v = \left[\frac{q}{C}\right]^+ \quad (14)$$
$$N_{t_i} = \frac{D_i}{q} \quad (15)$$
$$d_i = \begin{cases} d_c, if\ i = 5 \\ (0.7644\,Deg_i^2 + 0.2009 Deg_i + 0.0161) \cdot d_c,\ else \end{cases} \quad (16)$$

Notably, Equation 16 determines the average distance to be travelled from DCs to customers in different DN configurations, leveraging the formula introduced by Cantini et al. (2022b). Whereas Equations 17-19 depend on the inventory control policy adopted for the SKU, which is assumed to respect a reorder point (RP_i), safety stocks (SS_i), and an optimal order quantity (Q_i).

$$Q_i = \sqrt{\frac{2 \cdot D_i \cdot o}{h \cdot c}} \quad (17)$$
$$RP_i = D_i \cdot L + SS_i \quad (18)$$
$$SS_i = Z \cdot \sigma \cdot \sqrt{L \cdot \frac{N}{DC_i}} \quad (19)$$

Hence, the described mathematical model, linking each SKU to its most cost-effective stock deployment policy, appears influenced by 14 influential variables (see Table A1). However, existing literature offers limited insight into how these variables interact with each other and impact the optimal stock deployment policy. Conversely, the application of BT to the DN configuration problem can provide a deeper understanding of variables' interaction and their impact on C_{TOT_i}, thus assisting retailers in making informed decisions about the optimal stock deployment policy.

3. BT application for DN configuration

To apply BT to the DN configuration problem three steps were performed, following (Langhaar, 1951). In step 1, a mathematical relation of type $F(A_1, A_2, \dots, A_n) = 0$ was identified to describe the addressed problem (i.e., DN configuration) and link it to n=14 influential variables of (Cantini et al., 2022b)'s model. The identified mathematical relation was constituted by Equation 3. In step 2, the influential variables representing the problem were listed and associated with their k physical dimensions, resulting in the dimensional matrix in Table 1. Table 1 includes k=4 physical dimensions: time [T], quantity [Q], money [$], and length [L, i.e., distance]. These dimensions are in line with those suggested for an operations management problem by (Miragliotta, 2011; Vignaux, 2001). Notably, the numbers in Table 1 are the values of exponents (powers) by which each influential variable is related to the respective physical dimension.

Table 1: Dimensional matrix representing the DN configuration problem

	Physical dimensions			
Influential variables	[Q]	[T]	[$]	[L]
N	0	0	0	0
Deg_i	0	0	0	0
SL	0	0	0	0
$\bar{D}$	1	-1	0	0
σ	1	-1	0	0
b	0	0	1	0
l	0	1	0	0
c	-1	0	1	0
o	0	0	1	0
h	0	-1	0	0
d_c	0	0	0	1
t	0	0	1	-1
q	1	0	0	0
C	1	0	0	0

Finally, in step 3, a set of $n - k$=10 independent dimensionless groups were found through which the DN configuration problem (i.e., Equation 3) could be reformulated as $\Phi(\pi_1, \pi_2, \dots, \pi_{n-k}) = 0$. To define dimensionless groups, among the 14 influential variables, we selected a subset of $k = 4$ independent ones by applying the Rouché-Capelli theorem (Horbiychuk et al., 2021). Accordingly, we sought out 4 variables whose dimensional sub-matrix (derived by excluding all rows and columns from Table 2 except those corresponding to the selected 4 variables) had a non-zero determinant. Particularly, we selected the subset: $\bar{D}$, l, c, and t. Hence, a system of linear algebraic equations was established to determine the exponents (denoted with α) based on which to raise variables' physical dimensions and make their ratio dimensionless. In this way, for each influential variable, we found a dimensionless group (i.e., the dimensionless ratio), herein expressed with π. Below we provide an example to clarify how dimensionless groups were achieved. Considering the influential variable b, proper exponents α were identified to render Equation 20 dimensionless. This involved searching for the exponents that verified Equation 21, resulting in $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 1$, and $\alpha_4 = 0$. These exponents led to the dimensionless group π_1 in Equation 22.

$$\pi_1 = \frac{b}{\bar{D}^{\alpha_1} \cdot l^{\alpha_2} \cdot c^{\alpha_3} \cdot t^{\alpha_4}} \quad (20)$$

$$[\$] = [Q \cdot T^{-1}]^{\alpha_1} \cdot [T]^{\alpha_2} \cdot [\$ \cdot Q^{-1}]^{\alpha_3} \cdot [\$ \cdot L^{-1}]^{\alpha_4} \quad (21)$$

$$\pi_1 = \frac{b}{\bar{D} \cdot l \cdot c} \quad (22)$$

The approach shown for b was repeated for all influential variables, leading to the dimensionless groups in Equations 23-31. Moreover, this approach was applied to C_{TOT_i}, achieving the dimensionless group in Equation 32.

$$\pi_2 = N \quad (23)$$

$$\pi_3 = Deg_i \quad (24)$$

$$\pi_4 = SL \quad (25)$$

$$\pi_5 = \frac{\sigma}{\bar{D}} \quad (26)$$

$$\pi_6 = \frac{o}{\bar{D} \cdot l \cdot c} \quad (27)$$

$$\pi_7 = h \cdot l \quad (38)$$

$$\pi_8 = \frac{d_c \cdot t}{\bar{D} \cdot l \cdot c} \quad (29)$$

$$\pi_9 = \frac{q}{\bar{D} \cdot l} \quad (30)$$

$$\pi_{10} = \frac{C}{\bar{D} \cdot l} \quad (31)$$

$$\pi^* = \frac{C_{TOT_i}}{\bar{D} \cdot c} \quad (32)$$

In summary, BT facilitated the transformation of Equation 3 into a function of ten dimensionless groups, as shown in Equation 35. The following Section elaborates the practical utility of Equation 35.

$$\frac{C_{TOT_i}}{\bar{D} \cdot c} = \Phi\left(\frac{b}{\bar{D} \cdot l \cdot c}, N, Deg_i, SL, \frac{\sigma}{\bar{D}}, \frac{o}{\bar{D} \cdot l \cdot c}, h \cdot l, \frac{d_c \cdot t}{\bar{D} \cdot l \cdot c}, \frac{q}{\bar{D} \cdot l}, \frac{C}{\bar{D} \cdot l}\right) \quad (35)$$

4. Results and discussion

An immediate benefit of applying BT is the reduction of influential variables affecting DN configuration (i.e., from 14 affecting C_{TOT_i} in Equation 3 to 10 dimensionless groups affecting π^* in Equation 35). This reduction may seem insufficient for streamlining DN configuration choices and support retailers' decision-making. However, the usefulness of BT can be seen from another perspective. Dimensionless groups, by definition, enable similarity and scaling considerations for stock deployment policies. Plotting the variation of these dimensionless groups and their consequent impact on C_{TOT_i} of different stock deployment policies yields performance maps. In turn, performance maps offer two contributions. Firstly, they provide deeper insights into the decision-making problem by graphically representing the relationships between influential variables and their impacts on the cost performance of centralized, decentralized, and hybrid DNs. Secondly, they suggest similar stock deployment decisions across contexts with identical dimensionless group values. Therefore, retailers can optimize their stock deployment policies by referring to the optimal decisions indicated for similar DNs, eliminating the need to apply the mathematical formulas in Section 3. Accordingly, the decision-making process is simplified while maximizing the knowledge gained from the dimensionless family of DN configurations outlined in the performance maps.

While creating performance maps based on ten dimensionless groups might seem challenging, (Sonin, 2004) proposes an approach to accomplish this task. According to (Sonin, 2004), the number of dimensionless groups representing a problem can be reduced by fixing the values of influential variables over which decision-makers (i.e., retailers) have no control. Building on this concept, this paper presents an example of practical BT application on the DN configuration of a real company (Company A), providing the respective performance maps. Based on the performance maps, this paper explains how maps can be leveraged by retailers to make decisions and how their outcomes provide insights into DN configuration. We acknowledge that the provided example does not cover all possible applications of BT in DN configuration. Nevertheless, this study serves as a basis for exploring other applications in future works.

4.1 Case study application

Company A is an Italian bus spare parts retailer, which owns 5 DCs. Among the SKUs managed by this retailer, a specific one is served to a well-consolidated customer market (i.e., N, $\bar{D}$, σ, q, b, c, and h fixed). For this SKU, the retailer follows an established procedure for ordering stock replenishment from suppliers (o fixed). This retailer is interested in gaining knowledge on how the total cost of the existing DN (C_{TOT_i}) is affected by the choice of the suppliers to procure the SKU (upstream DCs) and the transportation mode for delivering the SKU to customers (downstream DCs). Moreover, the retailer is interested in evaluating if the current stock deployment policy (i.e., centralization in a single DC among the 5 ones) associated with the considered SKU with the current supplier and transportation mode is optimized or should be modified to reduce total logistic costs (switching to decentralized or hybrid DN configurations). Also, the retailer wants to compare the current supplier and transportation mode with novel alternatives (summarized in Table 2), aiming to assess if new distribution options could be more cost-effective and if the changes of suppliers and transportation mode should result in varying stock deployment decisions.

Table 2. Characteristics of current and new possible suppliers and transportation modes

Influential variable [unit measure]	Current combination supplier-transport mode	New possible combination supplier-transport mode
t [€/km*vehicle]	0.70	1.05
C [units]	5,000	7,500
l [years]	0.010	0.015

In this scenario, BT facilitates the creation of performance maps (Figure 2) suggesting the most cost-effective stock deployment policy ($\boldsymbol{Deg_i}$) to be associated with the considered SKU as the dimensionless groups vary, with all influential variables of Table A1 fixed except those linked to the characteristics of the supplier ($\boldsymbol{l}$) and transportation mode ($\boldsymbol{SL}$, $\boldsymbol{d_c}$, $\boldsymbol{t}$ and $\boldsymbol{C}$). By fixing all influential variables that are beyond the retailer's control (i.e., $\boldsymbol{N}$, $\boldsymbol{\overline{D}}$, $\boldsymbol{\sigma}$, $\boldsymbol{q}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{h}$, and $\boldsymbol{o}$), the problem's degrees of freedom are reduced, leading Equation 35 to transform into Equation 36.

$$C_{TOT_i} = \Phi\left(Deg_i, SL, \frac{d_c \cdot t}{l}, \frac{C}{l}\right) \tag{36}$$

Accordingly, we can derive performance maps (Figure 2) that provide insights on how suppliers' and transportation mode's characteristics affect the choice between centralized, decentralized, or hybrid DN configurations. As a matter of fact, the groups in Equation 36 are no longer dimensionless. However, according to (Sonin, 2004), they constitute the most efficient parameters for scaling DN configurations and allowing similitude considerations to be made in this application scenario.

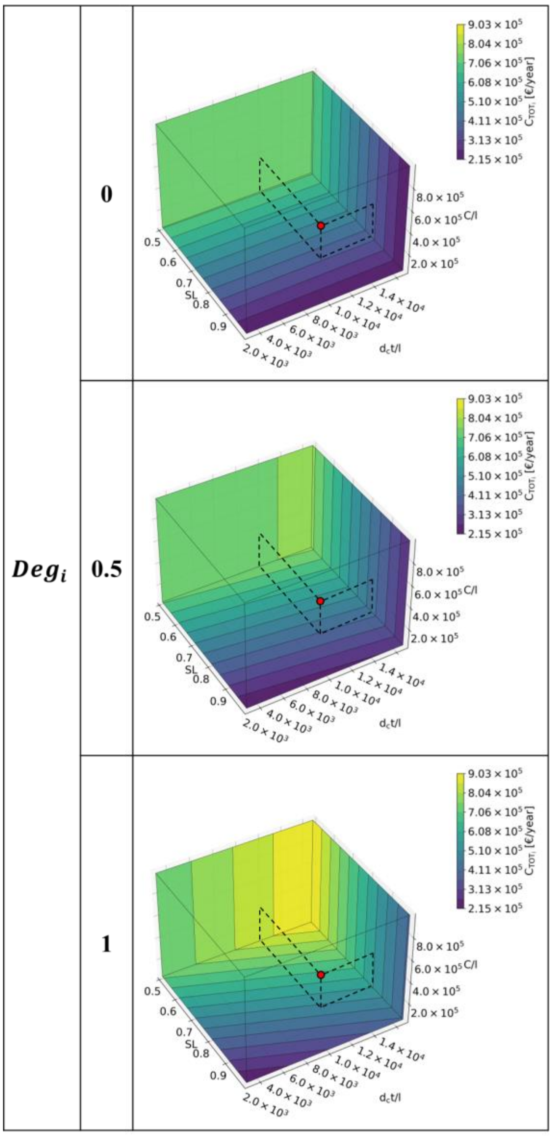

Figure 2. Performance maps for the proposed case study

4.2 Case study discussion

The main contribution of the performance maps in Figure 2 are as follows. First, for the considered SKU, they depict the mutual interaction between parameters characterizing suppliers and transportation mode, and their impact on the total DN configuration cost (as per Equation 36). Therefore, they provide the retailer with insights into the decision-making problem. For instance, considering a supplier and a transportation mode characterized by values of the dimensionless groups expressed by the red dot in Figure 2 (where the dashed lines show the dot projections on the x, y, and z axes), the retailer may note the following considerations. As the degree of centralization ($\boldsymbol{Deg_i}$) increases – namely sliding down in Figure 2 –, the total cost of the DN ($\boldsymbol{C_{TOT_i}}$) increases (i.e., the colour of curves changes from blue to yellow, with an increase in $\boldsymbol{C_{TOT_i}}$ of around 127%). This result proves that, for the considered SKU, the current stock deployment policy adopted (i.e., centralization, with $\boldsymbol{Deg_i}$=1) is not optimal. Rather, in the existing DN, the retailer should switch to decentralization ($\boldsymbol{Deg_i}$=0), keeping stocks of the SKU in all the 5 DCs owned by Company A.

Furthermore, within each performance map in Figure 2 (e.g., focusing on the lower one, which represents the current situation with $\boldsymbol{Deg_i}$=1), the total cost of the DN changes as follows. On the x-axis, $\boldsymbol{C_{TOT_i}}$ significantly increases as the service level ($\boldsymbol{SL}$) decreases. On the y-axis, $\boldsymbol{C_{TOT_i}}$ increases if the value of the dimensionless group $\frac{d_c \cdot t}{l}$ increases (namely, when the procurement lead time is lower than the product between the unitary transportation cost and the distance from DC to customers). Conversely, on the z-axis, $\boldsymbol{C_{TOT_i}}$ remains approximately constant if the value of the dimensionless group $\frac{C}{l}$ increases, meaning that the ratio between the capacity of transportation vehicles and the procurement lead time has lower influence on $\boldsymbol{C_{TOT_i}}$ compared with the other dimensionless groups. Since the performance maps are three-dimensional, this proves that dimensionless groups on the x and y axes ($\boldsymbol{SL}$ and $\frac{d_c \cdot t}{l}$) not only have marked impact on the cost performance of the current DN configuration but also have a strong mutual interaction when affecting the DN.

Another contribution of the proposed performance maps is that Figure 2 provides the retailer with user-friendly visual tools to conduct similarity and scaling considerations. Specifically, the retailer can associate all suppliers and transportation modes characterized by identical values of dimensionless groups (i.e., scaling parameters in Equation 36) to the same optimal stock deployment policies, eliminating the need to solve the mathematical model outlined in Section 2. For example, suppose the retailer plans to substitute its transportation mode and supplier by switching from the current alternatives to new ones with the characteristics in Table 2. Suppose both the current position of DCs and the customer service level remain unchanged (i.e., $\boldsymbol{d_c}$=100 km and $\boldsymbol{SL}$=0.95, respectively). In these conditions, with reference to Figure 2, the retailer can quickly identify the most cost-effective stock deployment policy to be associated with the new combination supplier-transportation mode by leveraging the knowledge already gained for the current combination. Indeed, according to the characteristics outlined in Table 2, the current and new alternatives share the same values of dimensionless groups (i.e., $\frac{d_c \cdot t}{l}$=7'000 and $\frac{C}{l}$=500'000). Therefore, they should be associated to the same optimal stock deployment policy (i.e., decentralization with Deg_i=1), meaning that, under similarity conditions, the current and new combinations of supplier-transportation mode show the same economic impact (C_{TOT_i}). This example illustrates how the performance maps provided in Figure 2 can assist the retailer in reducing the time and computational efforts required to make distribution and vendor-rating decisions, especially when evaluating several combinations of suppliers and transportation modes.

5. Conclusion

This paper proposes the application of BT to support retailers in configuring DNs, focusing on choosing the most cost-effective stock deployment policy (i.e., centralization, decentralization, or hybrid DN configurations). After modeling the DN configuration problem in a mathematical form, BT checks the meaningfulness of its governing equations, identifies influential variables, and extracts knowledge on how they interact with each other when affecting the most cost-effective stock deployment policies. Hence, BT facilitates retailers' informed decisions about the most cost-effective stock deployment to select. Moreover, BT allows for creating performance maps which suggest similar stock deployment decisions for scaled DNs.

At a theoretical level, this paper is the first one proposing BT (and, more in general DA) applications in the field of supply chain management. Moreover, the provided performance maps are the first visual tools proposed in the literature to summarize and explicitly compare the cost performance of centralized, decentralized, and hybrid SC configurations. At a practical level, the provided performance maps can reduce the time and computational efforts needed by retailers to optimize stock deployment policies. The findings of this paper (in particular the usefulness of the proposed performance maps) have been tested on an example case study. The case study has proven different contributions of the performance maps, allowing a retailer to evaluate: (i) the impact of different variables on the total cost (C_{TOT_i}) of the DN; (ii) the cost-effectiveness of changing the current stock deployment policy (switching from centralization to decentralization); (iii) and the possibility to change the current combination of supplier-transportation mode without impacting C_{TOT_i}, thus reviewing the supply chain actors.

The main limitations of this study are two. Firstly, BT is applied on the proposed mathematical model of (Cantini et al., 2022b), which is valid under certain initial assumptions. For instance, an (RP, Q) inventory control policy is assumed to manage stocks, SKUs are considered to follow a normal demand, etc. Secondly, a single case study is shown to test the proposed methodology, while BT application in DN configuration can open to many other applications. For example, we could consider the typical scenario described by (Cantini et al., 2022a) in which a retailer has well-consolidated partnership with a supplier (l fixed), possesses DCs at known geographical locations (d_c fixed), owns a fleet of distribution vehicles (t and C fixed), follows an established procedure for ordering stock replenishment from the supplier (o fixed), and serves a specific customer market by guaranteeing a service level (N, SL fixed). In this scenario, BT can facilitate the creation of performance maps suggesting the optimal stock deployment policy (Deg_i) to be associated with each SKU as the dimensionless groups in Equation 35 vary, with all influential variables of Table A1 fixed except those linked to the characteristics of the SKU and customer demand (i.e., b, c, h, $\overline{D}$, σ, and q). Accordingly, the provided performance maps would result in visual tools for inventory classification, associating groups of similar SKUs to similar DN configuration decisions. To remove the aforementioned limitations, future developments of this study could be two. First, to repeat the BT application when relaxing some simplifying model assumptions. For instance, considering SKUs with other demand distributions (e.g., Poisson), stochastic lead time, other inventory control policies, etc. Finally, to deepen the BT application in case studies and in other scenarios rather than the one described in Section 4.1. Eventually, BT applications could also be investigated in other operations management problems, providing other useful performance maps for assisting companies' decision-making.

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Appendix A. NOTATION

Table A1: Nomenclature used in this paper

Index	Description	Unit measure
i	Stock deployment policy (i=1, 2, 3)	-
Influential variable (input data)	**Description**	**Unit measure**
N	Number of customers served	-
Deg_i	Degree of centralization (0, 0.50, 1)	-
SL	Expected service level (associated with the service factor Z in a standard normal distribution)	-
$\bar{D}$	Mean demand of one customer for the considered SKU	units/time
σ	Standard deviation of the demand of one customer	units/time
b	Unitary backorder cost of the SKU	€/backorder
L	Average procurement lead time for receiving the SKU by the supplier	time
c	Unitary cost of purchasing the SKU from the supplier	€/unit
o	Cost of issuing a supply order	€/order
h	Inventory holding cost rate	time^{-1}
d_c	Average distance from central DC to customers (when $i = 5$)	km
t	Unitary transportation cost	€/km*vehicle
q	Average quantity of SKU ordered by a customer in each demand	units/demand
C	Capacity of transport vehicle	units
Decision variable	**Description**	**Unit measure**
Q_i	Optimal reorder quantity of the SKU in a DC	units
RP_i	SKU reorder point in a DC	units
SS_i	SKU safety stocks in a DC	units
DC_i	Number of DCs in the DN	-
D_i	Annual demand received by all customers in a DC	units/time
N_v	Average number of vehicles to transport the SKU from a DC to customers	vehicles/transportation
N_{t_i}	Average number of transports performed to distribute the SKU from a DC to customers	transportations/time
d_i	Average distance from a DC to customers	km
I_i	Average inventory level in a DC	units
N_{o_i}	Average number of supply orders for the SKU	orders/time
N_{b_i}	Average number of backorders in a DC	orders/time
Evaluated cost	**Description**	**Unit measure**
C_{TOT_i}	Total cost of the considered DN configuration	€/time
C_{P_i}	Annual purchase cost	€/time
C_{H_i}	Annual holding cost	€/time
C_{O_i}	Annual ordering cost	€/time
C_{B_i}	Annual backorder cost	€/time
C_{T_i}	Annual transportation cost	€/time